Project number: 2018-1-IT02-KA201-048274

INTERVENTION TOOL

Simplification of Algebraic Expressions

1. Introduction

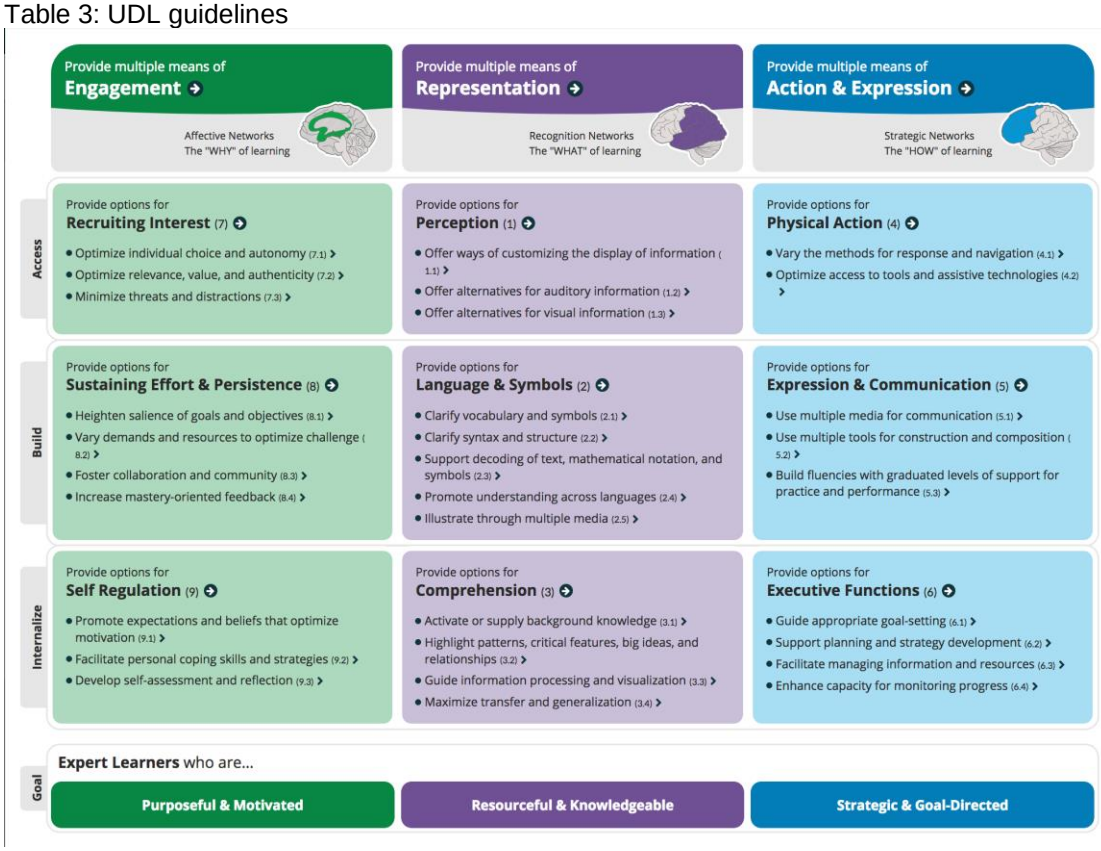
In order to develop a set of educational activities aimed to simplify algebraic expressions with addition/subtraction of monomials, we refer to some theoretical frameworks that will be described in the session 2.
In session 3 the design of the educational activities is described. In particular, if the activities are addressed to a single student, to small groups or to the class, the educational aim of the activities, the cognitive area and math domain of interest and the Mathematical objects in the areas of difficulties identified through the B2 questionnaire.

2. Theoretical framework of reference

The theoretical references that helped us to design the following activities are:

1) **Universal design for learning (UDL) principles** (Table 3), a framework specifically conceived to design *inclusive* educational activities (http://udlguidelines.cast.org/)

Table 3: UDL guidelines



The Center for Applied Special Technology (CAST) has developed a comprehensive framework around the concept of Universal Design for Learning (UDL), with the aim of focusing research, development, and educational practice on understanding diversity and facilitating learning (Edyburn, 2005). UDL includes a set of Principles, articulated in *Guidelines and Checkpoints*[1]. The research grounding UDL's framework is that "learners are highly variable in their response to instruction. [...]"

[1] For a complete list of the principles, guidelines and checkpoints and a more extensive description of CAST's activities, visit http://www.udlcenter.org

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Thus, UDL focus on these individual differences as an important element to understanding and designing effective instruction for learning.
To this aim, UDL advances three foundational Principles: 1) provide multiple means of representation, 2) provide multiple means of action and expression, 3) provide multiple means of engagement. In particular, guidelines within the first principle have to do with means of perception involved in receiving certain information, and of "comprehension" of the information received. Instead, the guidelines within the second principle take into account the elaboration of information/ideas and their expression. Finally, the guidelines within the third principle deal with the domain of "affect" and "motivation", also essential in any educational activity.
For our analyses we will focus in particular on specific guidelines within the three Principles[2].
Guidelines within Principle 1 (provide multiple means of representation), suggest proposing different options for perception and offering support for decoding mathematical notation and symbols. Moreover, guidelines suggest the importance of providing options for comprehension highlighting patterns, critical features, big ideas, and relationships among mathematical notions. Finally, our analyses will give examples of how software AlNuSet can guide information processing, visualization, and manipulation, in order to maximize transfer and generalization.
Moreover, the guidelines from Principle 2 (provide multiple means of action and expression) suggest to offer different options for expression and communication supporting planning and strategy development. Finally, the guidelines from Principle 3 show how certain activities can recruit students' interest, optimizing individual choice and autonomy, and minimizing threats and distractions.
In the section 4 we will analyse examples of activities, classifying them both by the type of mathematical learning they are designed and the cognitive area they support. We will show how these examples have been designed on the UDL principles in order to make them inclusive and effective to overcame math difficulties identified through B2 questionnaire.

2) The European Project **FasMed,** that focused on formative assessment in mathematics and science, (https://research.ncl.ac.uk/fasmed/).
Formative assessment (FA) is conceived as a method of teaching where "evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited" (Black & Wiliam, 2009, p. 7).
FaSMEd project refers to Wiliam and Thompson (2007)'s study, that identifies five key strategies for FA practices in school setting: (a) *clarifying and sharing learning intentions and criteria for success;* (b) *engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding;* (c) *providing feedback that moves learners forward;* (d) *activating students as instructional resources for one another;*–(e) *activating students as the owners of their own learning.*
The teacher, student's peers and the student him- or herself are the agents that activate these FA strategies.

Table 4: Formative assessment strategies

<table>
<tr><th></th><th>Where the learner is going</th><th>Where the learner is right now</th><th>How to get there</th></tr>
<tr><td>Teacher</td><td rowspan="3">1 Clarifying learning intentions and criteria for success</td><td rowspan="2">2 Engineering effective class-room discussions and other learning tasks that elicit evidence of student understanding</td><td rowspan="2">3 Providing feedback that moves learners forward</td></tr>
<tr><td>Peer</td></tr>
<tr><td>Learner</td><td colspan="2">4 Activating students as instructional resources for one another</td></tr>
<tr><td></td><td>Understanding and sharing learning intentions and criteria for success</td><td colspan="2">5 Activating students as the owners of their own learning</td></tr>
<tr><td></td><td>Understanding learning intentions and criteria for success</td><td colspan="2"></td></tr>
</table>

[2] The items are taken from the interactive list at http://www.udlcenter.org/research/researchevidence

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FaSMEd activities are organized in sequences, that encompass group work on worksheets and class discussion where selected group works are discussed by the whole class, under the orchestration of the teacher. Taking into account formative assessment strategies and technology functionalities, Cusi, Morselli & Sabena (2017, p. 758) designed three types of worksheets for the classroom activity:

> "(1) *problem worksheets:* worksheets introducing a problem and asking one or more questions involving the interpretation or the construction of the representation (verbal, symbolic, graphic, tabular) of the mathematical relation between two variables (e.g. interpreting a time-distance graph);
>
> (2) *helping worksheets*, aimed at supporting students who face difficulties with *the problem worksheets* by making specific suggestions (e.g. guiding questions);
>
> (3) *poll worksheets*: worksheets prompting a poll among proposed options".

The authors identified feedback strategies (Table 5) the teacher may adopt to give feedback to students (Cusi, Morselli & Sabena, 2018, p. 3466). These strategies are employed in the class discussion that is organized by the teacher after the group work on worksheets.

Table 5:

Revoicing	When the teacher mirrors one student's intervention so as to draw the attention on it. Often, during the revoicing, the teacher stresses with voice intonation some crucial words of the sentence she is mirroring. Rephrasing takes place when the teacher reformulates the intervention of one student, with the double aim of drawing the attention of the class and making the intervention more intelligible to everybody.
Rephrasing	Rephrasing takes place when the teacher reformulates the intervention of one student, with the double aim of drawing the attention of the class and making the intervention more intelligible to everybody. Rephrasing is applied when the teacher feels that the intervention could be useful but needs to be communicated in a better way so as to become a resource for the others. […] The revoicing and rephrasing strategies […] turn one student (the author of the intervention) into a resource for the class.
Rephrasing with scaffolding	When the teacher, besides rephrasing, adds some elements to guide the students' work.
Relaunching	When the teacher reacts to a student's intervention, which (s)he considers interesting for the class, not giving a direct feedback, but posing a connected question. In this way, by relaunching the teacher provides an implicit feedback […] on the student's intervention, suggesting that the issue is interesting and worth to be deepened or, conversely, has some problematic points and should be reworked on.
Contrasting	Contrasting takes place when the teacher draws the attention on two or more interventions, representing two different positions, so as to promote a comparison. By contrasting, […] the authors of the two positions may be resource for the class as well as responsible of their own learning.

We draw from the FaSMEd experience the idea of creating classroom activities in the formative assessment perspective, which may promote inclusion.

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3. Design

3.1. Difficulties identified through the B2 questionnaire

We detect difficulties in the following item of B2:

1. Simplify:

a) a × a = ___

b) a + a = ___

c) 2a : a = ___

d) $\frac{2a}{a}$ = ____

e) $\sqrt[2]{a^3} = a^{-}$

f) a + 2a = _____

g) a×(b+c)= _____

These difficulties are related to the simplification of algebraic expressions.

3.2. Cognitive area and math domain of interest

The area of difficulties identified through the B2 questionnaire is related to the domain of *Algebra*. In particular, the difficulties are related to the simplification of algebraic expressions. Thus, *Memory* and *Visuo-Spatial* are the cognitive areas involved.

3.3. Educational Aims

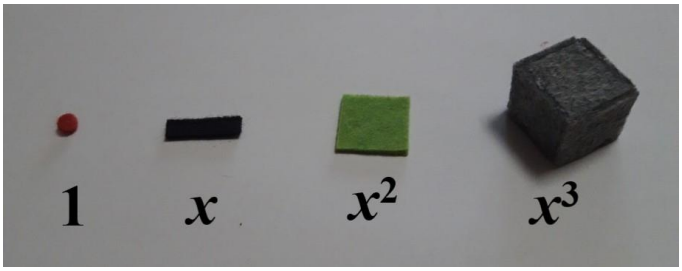
The intervention tool is aimed to simplify algebraic expressions with addition/subtraction of monomials, using manipulable materials.

3.4. Addressing to Student /class

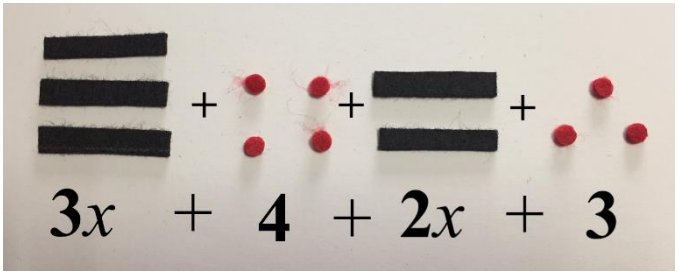
The intervention tool is to be addressed to small groups.

3.5. Educational activities: the Intervention Tool

The teacher introduces students to the materials to use and their symbology:



Then, the teacher writes the algebraic expression $3x + 4 + 2x + 3$ on the board and asks students to represent the same expression, using the materials and the symbology.



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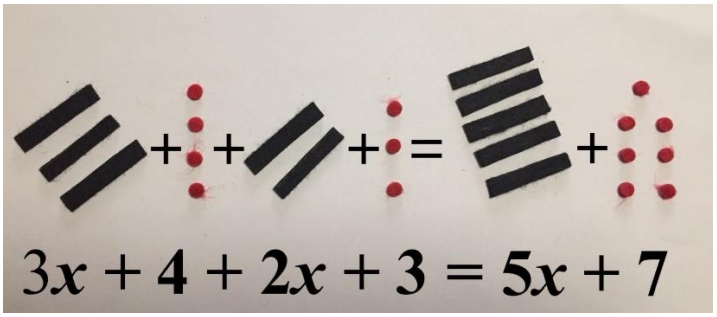
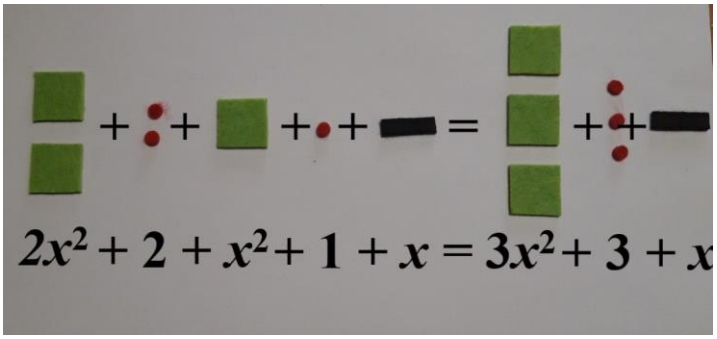
After observing the set of symbols, the student should write the mathematical expression and simplify it.



The same process will be used to simplify other algebraic expressions, such as

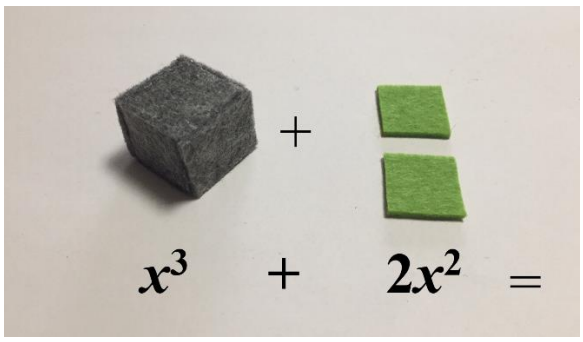
$$2x^2 + 2 + x^2 + 1 + x$$



Students can simplify more expressions using the material:

- $2x^2 + 1 + x^2 + 4$
- $2x^3 + x^2 + 3x + 1 + 2x^2 + 3$

After students have simplified the previous expressions, the teacher presents a new expression and asks students to simplify $x^3 + 2x^2$.



Once again, using the symbology, students should conclude that it is not possible to simplify the given expression.

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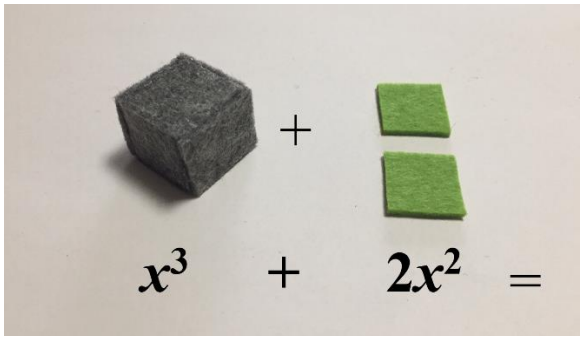
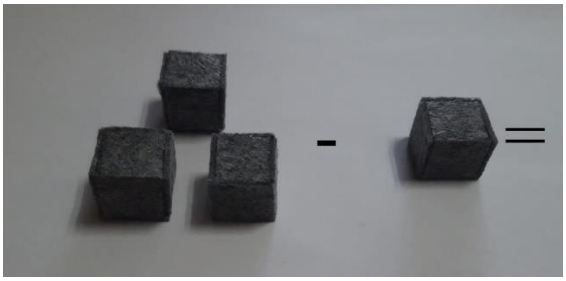


Finally the teacher challenges students to simplify the expression $3x^3 - x^3$.



4. References

[1] Karagiannakis, G. N., Baccaglini-Frank, A. E., & Roussos, P. (2016). Detecting strengths and weaknesses in learning mathematics through a model classifying mathematical skills. Australian J. of Learning Difficulties, 21(2), 115–141. \

[2] Workshop with Dr. Giannis Karagianakis in International Congress of math learning difficulties, in Lisbon, 7,8/02/2020

[3] Black, P., & Wiliam, D. (2009). Developing the theory of formative assessment. Educational Assessment, Evaluation and Accountability, 21(1), 5-31.

[4] Black, P., & Wiliam, D. (1998). Assessment and classroom learning. Assessment in education, 5(1), 7-74.

[5] Wiliam, D. (1999). Formative assessment in mathematics Part 2: feedback. Equals: Mathematics and Special Educational Needs, 5(3), 8-11

[6] European Project FasMed (https://research.ncl.ac.uk/fasmed/)

[7] Universal design for learning (UDL) principles (http://udlguidelines.cast.org/)

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