Project Number: 2018-1-IT02-KA201-048274

INTERVENTION TOOL

Supporting Memory in Remembering Rules of Signs in Multiplication Integer Numbers

University of Genova[1]

1. Introduction

In order to develop educational activities aimed to support memory in arithmetic, we refer to some theoretical frameworks that will be described in the session 2.

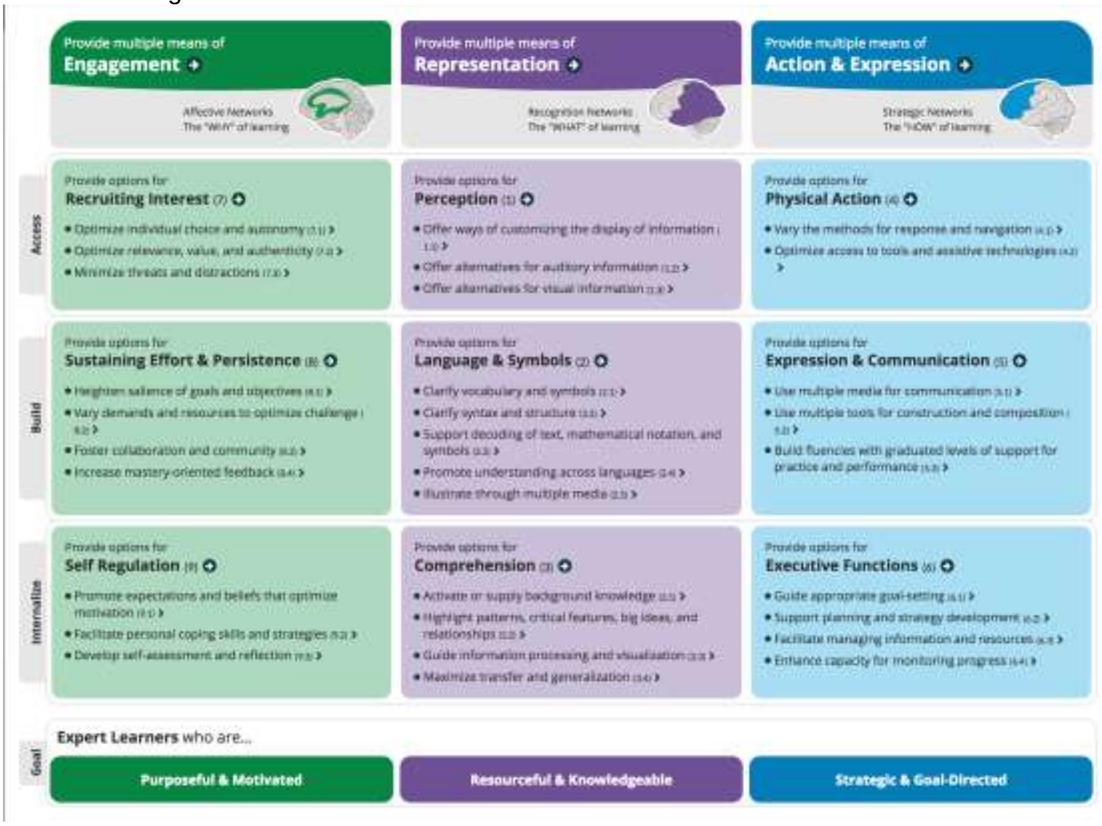
In session 3 the design of the educational activities is described. In particular, if the activities are addressed to a student or the class, the educational aim of the activities, the Cognitive area and math domain of interest and the Mathematical objects in the areas of difficulties identified through the B2 questionnaire

2. Theoretical framework of reference

The theoretical references that helped us to design the following activities are:

1) **Universal design for learning (UDL) principles** (Table 3), a framework specifically conceived to design *inclusive* educational activities (http://udlguidelines.cast.org/)

Table 3: UDL guidelines



[1] Emanuela De Negri, Elisabetta Robotti, Francesca Morselli, Paola Viterbori, Anna Siri, Laura Capelli

Co-funded by the Erasmus+ Programme of the European Union
The European Commission support for the production of this publication does not constitute an endorsement of the contents which reflects the views only of the authors, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

The Center for Applied Special Technology (CAST) has developed a comprehensive framework around the concept of Universal Design for Learning (UDL), with the aim of focusing research, development, and educational practice on understanding diversity and facilitating learning (Edyburn, 2005). UDL includes a set of Principles, articulated in *Guidelines and Checkpoints*[2]. The research grounding UDL's framework is that "learners are highly variable in their response to instruction. [...]"
Thus, UDL focus on these individual differences as an important element to understanding and designing effective instruction for learning.
To this aim, UDL advances three foundational Principles: 1) provide multiple means of representation, 2) provide multiple means of action and expression, 3) provide multiple means of engagement. In particular, guidelines within the first principle have to do with means of perception involved in receiving certain information, and of "comprehension" of the information received. Instead, the guidelines within the second principle take into account the elaboration of information/ideas and their expression. Finally, the guidelines within the third principle deal with the domain of "affect" and "motivation", also essential in any educational activity.
For our analyses we will focus in particular on specific guidelines within the three Principles[3].
In order to characterize students' difficulties in geometry, we refer to the following elements of Karagiannakis' and colleagues' frame (Table 1), which dealt with Memory in retrieval of geometrical facts and geometrical processing: retrieval geometrical facts, remembering theorems, remembering hypothesis and thesis which are focusing on.

Table 1: Karagiannakis's and colleagues' frame: domains of the four-pronged model and sets of mathematical skills associated with each domain

Domain	Mathematical skills associated with the domain
Core number	Estimating accurately a small number of objects (up to 4); estimating approximately quantities; placing numbers on number lines; managing Arabic symbols; transcoding a number from one representation to another (analogical-Arabic-verbal); counting principles awareness
Memory (retrieval and processing)	Retrieving numerical facts; decoding terminology (numerator, denominator, isosceles, equilateral); remembering theorems and formulas; performing mental calculations fluently; remembering procedures and keeping track of steps
Reasoning	Grasping mathematical concepts, ideas and relations; understanding multiple steps in complex procedures/algorithms; grasping basic logical principles (conditionality – "if … then …" statements – commutativity, inversion); grasping the semantic structure of problems; (strategic) decision-making; generalizing
Visual-spatial	Interpreting and using spatial organization of representations of mathematical objects (for example, numbers in decimal positional notation, exponents, geometrical 2D and 3D figures or rotations); placing numbers on a number line; confusing Arabic numerals and mathematics symbols; performing written calculation when position is important (e.g. borrowing/carrying); interpreting graphs and tables

3. Design

3.1 Difficulties identified through the B2 questionnaire

We detect difficulties in the following item of B2:
$(-2) \times (-3) = \ldots$
$(-12) \times (23) = \ldots$

Difficulties are related to recover from memory of rules of sign in multiplication of relative numbers.

[2] For a complete list of the principles, guidelines and checkpoints and a more extensive description of CAST's activities, visit http://www.udlcenter.org
[3] The items are taken from the interactive list at http://www.udlcenter.org/research/researchevidence

3.2 Cognitive area and math domain of interest

The area of difficulties identified through the B2 questionnaire is related to the domain of *Arithmetic*. *Memory* is the cognitive area involved.
In Table 1 the location of difficulties with respect to cognitive domain and mathematical area.

Table 1: The difficulties detected are linked to the cognitive domain of *Memory* and in the domain of *Arithmetic*

	Arithmetic	Geometry	Algebra
Memory	(-2) × (-3) = … (–12) × (23) = …		
Reasoning			
Visuo-spatial			

3.3 Educational Aims

The intervention tool is aimed at visualizing the multiplication operation between relative numbers, through a geometrical dynamic model available on AlNuSet. It promotes understanding of the rules of the sign.

3.4 Addressing to Student /class

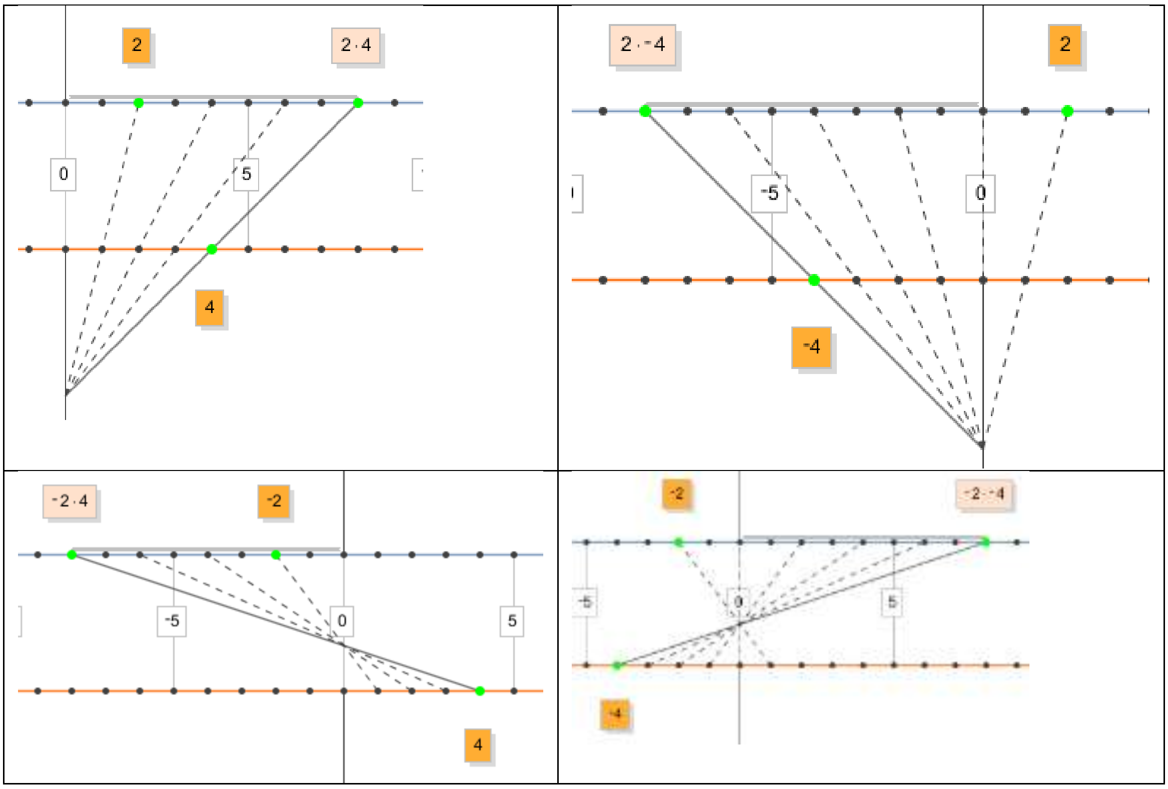
The Intervention tool is an educational activity that can to be carried out with all the students of the class.

3.5 Educational activities: the Intervention Tool

When introducing the multiplication operation between relative numbers, there is the educational need for a model that can support understanding of the rules of the sign in this operation.
To this aim, we consider the Algebraic Line environment of the software AlNuSet.
In particular we use the geometric model of multiplication, available in this environment.
We note that quantities having opposite sign can be represented by lines drawn in the opposite direction with respect to 0 (for example quantity 2 and quantity -2).
The geometric model allows multiplying a quantity (for example, the quantity 2 or -2, as in the figure) according to values with opposite sign (+4 or -4) by carrying this quantity with respect to these values on the straight line.
In Table 1 are four examples referring to the four cases of multiplication in this numerical set.

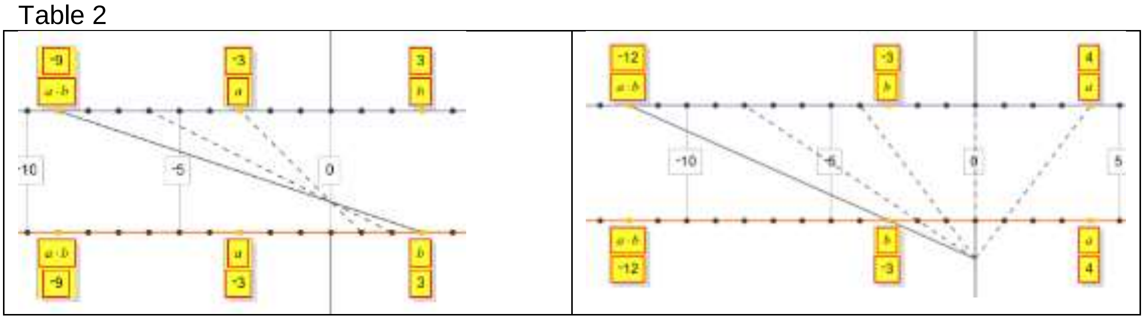
Co-funded by the Erasmus+ Programme of the European Union
The European Commission support for the production of this publication does not constitute an endorsement of the contents which reflects the views only of the authors, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

Table 1



In the first image, the model was used to calculate 2 · 4. The quantity 2 is multiplied for 4, i.e. reported according to the value 4 (4 times) on the straight line. The result is clearly positive.
In the second image, the model was used to calculate 2 · -4. The quantity 2 is multiplied by -4, that is, it is reported according to the value -4 on the straight line. The result is clearly negative.
In the third image, the model was used to calculate -2 · 4. The algebraic quantity -2 is multiplied for 4, that is, it is reported according to the value 4 on the straight line. The result is clearly negative.
In the fourth image we used the model to calculate -2 · -4. The algebraic quantity -2 is multiplied by -4, that is, it is reported according to the value -4 on the straight line. The result is clearly positive.
A though on the functioning of the rules of multiplication can also be realized in a other way, inserting on the Algebraic line the variable points a and b and their product a X b, visualizing the geometric construction of this operation and moving the points a and b along the straight line.
In the following table2 you can visualize that.

Table 2



If the model is used to instantiate the division operation, it is easy to realize that this operation is not closed in the set of relative integers. In fact, the result of the operation is represented on the straight line only if the dividend is a multiple of the divisor.

Co-funded by the Erasmus+ Programme of the European Union

Project Number: 2018-1-IT02-KA201-048274

As for the sign of the division result, the use of the model allows us to understand that it follows the same rules as multiplication.

You can find more et detailed activities here:
http://www.alnuset.com/en/home

Discussion through UDL guidelines about the above-mentioned activities
In *red* our comments to illustrate the connection between the principles of UDL and our activities.

Table 3: Analysis of the activities through the Table of UDL principles.

Engagement	***Representation***	***Action & Expression***
Recruiting interest *Optimize individual choice and autonomy* *Minimize threats and distractions*	***Perception*** Offer ways of customizing the display of information Offer alternatives for auditory information Offer alternatives for visual information Different registers through which information are displayed (visual non verbal, verbal and symbolic)	***Physical Action*** Vary the methods for response and navigation Optimize access to tools and assistive technologies *AlNuSet allows students physical action on the algebraic objects and gives them appropriate feedbacks on their action*
Sustaining effort Persistence *Heighten salience of goals and objectives* *Vary demands and resources to optimize challenge* Foster collaboration and community *Increase mastery-oriented feedback* *Feedbacks of software support engagement and motivation with respect the elaboration of the solution*	***Language & Symbols*** Clarify vocabulary and symbols Clarify syntax and structure Offer alternative language and symbols to decode information and to work on the information *This is promoted by the use of different registers of representation: figural non verbal on the drawing, dynamic, colours* Support decoding of text, mathematical notation, and symbols	***Expression Communication*** Use multiple media for communication Use multiple tools for construction and composition Build fluencies with graduated levels of support for practice and performance To use different registers in order to communicate

of the task	Promote understanding across languages Illustrate through multiple media *This is promoted by the use of dynamic algebraic software such as AlNuSet.* Support decoding of text, math notation and symbols *This is promoted by the visualization of results of computation by dynamic drawing realized by AlNuSet*	
Self Regulation *Promote expectations and beliefs that optimize motivation* *Facilitate personal coping skills and strategies* Develop self-assessment and reflection	***Comprehension*** Activate or supply background knowledge Highlight patterns, critical features, big ideas, and relationships (checkpoint 3.2) Guide information processing and visualization Maximize transfer and generalization *To support generalization, the tasks suggest to visualize drawings on AlNuSet. Indeed, the drag function of AlNuSet allows students to identify invariants in the rules of signs and memorize them.* Perception, language and symbols, comprehension (Constructing useable knowledge, knowledge that is accessible for future decision-making, depends	***Executive functions*** *Guide appropriate goal-setting* *The dynamic visualization of invariant elements in the rules of signs allows student to manage executive functions: supporting memory s/he can focus on the reasons of the rule.* *Facilitate managing information and resources* *Enhance capacity for monitoring progress*

	not upon merely perceiving information, but upon active “information processing skills”)	

4. References

[1] Karagiannakis, G. N., Baccaglini-Frank, A. E., & Roussos, P. (2016). Detecting strengths and weaknesses in learning mathematics through a model classifying mathematical skills. Australian J. of Learning Difficulties, 21(2), 115–141.
[2] UDL Principles: http://udlguidelines.cast.org/
[3] AlNuSet: www.alnuset.com