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INTERVENTION TOOL

Functions

1. Introduction

The intervention tool is conceived to address specific difficulties related to the mathematical domain of algebra and the cognitive domain of reasoning. By means of the intervention tool, that is conceived for all the class, the students may reflect on the expression depending on a variable and on the connection between algebraic formula and graph.

The intervention tool exploits an existing resource, available on the web:
https://teacher.desmos.com/activitybuilder/custom/566b31734e38e1e21a10aac8

The resource is developed by means of the dynamic geometry software Desmos and is inserted into the section "Classroom activities". The resource contains student activities as well as a teacher guide.

2. Theoretical framework of reference

We recall here Karagiannakis's and colleagues' frame (Table 1), which helps to characterize students' difficulties in mathematics.

Table 1: Karagiannakis's and colleagues' frame: domains of the four-pronged model and sets of mathematical skills associated with each domain

Domain	Mathematical skills associated with the domain
Core number	Estimating accurately a small number of objects (up to 4); estimating approximately quantities; placing numbers on number lines; managing Arabic symbols; transcoding a number from one representation to another (analogical-Arabic-verbal); counting principles awareness
Memory (retrieval and processing)	Retrieving numerical facts; decoding terminology (numerator, denominator, isosceles, equilateral); remembering theorems and formulas; performing mental calculations fluently; remembering procedures and keeping track of steps
Reasoning	Grasping mathematical concepts, ideas and relations; understanding multiple steps in complex procedures/algorithms; grasping basic logical principles (conditionality – "if … then …" statements – commutativity, inversion); grasping the semantic structure of problems; (strategic) decision-making; generalizing
Visual-spatial	Interpreting and using spatial organization of representations of mathematical objects (for example, numbers in decimal positional notation, exponents, geometrical 2D and 3D figures or rotations); placing numbers on a number line; confusing Arabic numerals and mathematics symbols; performing written calculation when position is important (e.g. borrowing/carrying); interpreting graphs and tables

We also recall that, when constructing B2, we chose questions that were related to the cognitive areas as well to three mathematical domains: arithmetic, geometry, algebra (Core number is not related to all cognitive areas). As a result, we proposed questions that were located in some cells of the following table (Table 2):

Table 2: Double relation between cognitive areas (memory, reasoning and visuo-spatial) and mathematical domains (arithmetic, geometry, algebra).

	Arithmetic	Geometry	Algebra
Memory			
Reasoning			
Visuo-spatial			

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Here we present additional theoretical references that helped us to design the intervention tools.
First of all, we refer to the Universal design for learning (UDL) principles (Table 3), a framework specifically conceived to design inclusive educational activities (http://udlguidelines.cast.org/)

Table 3: UDL guidelines

	Provide multiple means of **Engagement**	Provide multiple means of **Representation**	Provide multiple means of **Action & Expression**
	Affective Networks The "WHY" of learning	Recognition Networks The "WHAT" of learning	Strategic Networks The "HOW" of learning
Access	Provide options for **Recruiting Interest** (7) • Optimize individual choice and autonomy (7.1) • Optimize relevance, value, and authenticity (7.2) • Minimize threats and distractions (7.3)	Provide options for **Perception** (1) • Offer ways of customizing the display of information (1.1) • Offer alternatives for auditory information (1.2) • Offer alternatives for visual information (1.3)	Provide options for **Physical Action** (4) • Vary the methods for response and navigation (4.1) • Optimize access to tools and assistive technologies (4.2)
Build	Provide options for **Sustaining Effort & Persistence** (8) • Heighten salience of goals and objectives (8.1) • Vary demands and resources to optimize challenge (8.2) • Foster collaboration and community (8.3) • Increase mastery-oriented feedback (8.4)	Provide options for **Language & Symbols** (2) • Clarify vocabulary and symbols (2.1) • Clarify syntax and structure (2.2) • Support decoding of text, mathematical notation, and symbols (2.3) • Promote understanding across languages (2.4) • Illustrate through multiple media (2.5)	Provide options for **Expression & Communication** (5) • Use multiple media for communication (5.1) • Use multiple tools for construction and composition (5.2) • Build fluencies with graduated levels of support for practice and performance (5.3)
Internalize	Provide options for **Self Regulation** (9) • Promote expectations and beliefs that optimize motivation (9.1) • Facilitate personal coping skills and strategies (9.2) • Develop self-assessment and reflection (9.3)	Provide options for **Comprehension** (3) • Activate or supply background knowledge (3.1) • Highlight patterns, critical features, big ideas, and relationships (3.2) • Guide information processing and visualization (3.3) • Maximize transfer and generalization (3.4)	Provide options for **Executive Functions** (6) • Guide appropriate goal-setting (6.1) • Support planning and strategy development (6.2) • Facilitate managing information and resources (6.3) • Enhance capacity for monitoring progress (6.4)
Goal	**Expert Learners** who are... **Purposeful & Motivated**	**Resourceful & Knowledgeable**	**Strategic & Goal-Directed**

The Center for Applied Special Technology (CAST) has developed a comprehensive framework around the concept of Universal Design for Learning (UDL), with the aim of focusing research, development, and educational practice on understanding diversity and facilitating learning (Edyburn, 2005). UDL includes a set of Principles, articulated in Guidelines and Checkpoints1. The research grounding UDL's framework is that "learners are highly variable in their response to instruction. [...]" Thus, UDL focus on these individual differences as an important element to understanding and designing effective instruction for learning.
To this aim, UDL advances three foundational Principles : 1) provide multiple means of representation, 2) provide multiple means of action and expression, 3) provide multiple means of engagement. In particular, guidelines within the first principle have to do with means of perception involved in receiving certain information, and of "comprehension" of the information received. Instead, the guidelines within the second principle take into account the elaboration of information/ideas and their expression. Finally, the guidelines within the third principle deal with the domain of "affect" and "motivation", also essential in any educational activity.

Furthermore, we refer to the experience of the European Project FasMed, that focused on formative assessment in mathematics and science, (https://research.ncl.ac.uk/fasmed/).
Formative assessment (FA) is conceived as a method of teaching where "evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited" (Black & Wiliam, 2009, p. 7). FaSMEd project refers to Wiliam and Thompson (2007)'s study, that identifies five key strategies for

[1] For a complete list of the principles, guidelines and checkpoints and a more extensive description of CAST's activities, visit http://www.udlcenter.org

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FA practices in school setting: (a) clarifying and sharing learning intentions and criteria for success; (b) engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding; (c) providing feedback that moves learners forward; (d) activating students as instructional resources for one another; (e) activating students as the owners of their own learning. The teacher, student's peers and the student him- or herself are the agents that activate these FA strategies.

Table 4: Formative assessment strategies

	Where the learner is going	Where the learner is right now	How to get there
Teacher	1 Clarifying learning intentions and criteria for success	2 Engineering effective class-room discussions and other learning tasks that elicit evidence of student understanding	3 Providing feedback that moves learners forward
Peer	Understanding and sharing learning intentions and criteria for success	4 Activating students as instructional resources for one another	
Learner	Understanding learning intentions and criteria for success	5 Activating students as the owners of their own learning	

FaSMEd activities are organized in sequences, that encompass group work on worksheets and class discussion where selected group works are discussed by the whole class, under the orchestration of the teacher. Taking into account formative assessment strategies and technology functionalities, Cusi, Morselli & Sabena (2017, p. 758) designed three types of worksheets for the classroom activity:

> "(1) problem worksheets: worksheets introducing a problem and asking one or more questions involving the interpretation or the construction of the representation (verbal, symbolic, graphic, tabular) of the mathematical relation between two variables (e.g. interpreting a time-distance graph);
>
> (2) helping worksheets, aimed at supporting students who face difficulties with the problem worksheets by making specific suggestions (e.g. guiding questions);
>
> (3) poll worksheets: worksheets prompting a poll among proposed options".

The authors identified feedback strategies (Table 5) the teacher may adopt to give feedback to students (Cusi, Morselli & Sabena, 2018, p. 3466). These strategies are employed in the class discussion that is organized by the teacher after the group work on worksheets.

Table 5:

Revoicing	When the teacher mirrors one student's intervention so as to draw the attention on it. Often, during the revoicing, the teacher stresses with voice intonation some crucial words of the sentence she is mirroring. Rephrasing takes place when the teacher reformulates the intervention of one student, with the double aim of drawing the attention of the class and making the intervention more intelligible to everybody.
Rephrasing	Rephrasing takes place when the teacher reformulates the intervention of one student, with the double aim of drawing the attention of the class and making the intervention more intelligible to everybody. Rephrasing is applied when the teacher feels that the intervention could be useful but needs to be communicated in a better way so as to become a resource for the others. […] The revoicing and rephrasing strategies […] turn one student (the author of the intervention) into a resource for the class.
Rephrasing with	When the teacher, besides rephrasing, adds some elements to guide the students' work.

scaffolding	
Relaunching	When the teacher reacts to a student's intervention, which (s)he considers interesting for the class, not giving a direct feedback, but posing a connected question. In this way, by relaunching the teacher provides an implicit feedback […] on the student's intervention, suggesting that the issue is interesting and worth to be deepened or, conversely, has some problematic points and should be reworked on.
Contrasting	Contrasting takes place when the teacher draws the attention on two or more interventions, representing two different positions, so as to promote a comparison. By contrasting, […] the authors of the two positions may be resource for the class as well as responsible of their own learning.

3. Design

3.1 Difficulties identified through the B2 questionnaire

The intervention tool aims at addressing specific difficulties that were outlined by means of Questionnaire B2, namely difficulties referring the meaning of variable and expression depending on a variable (see items Q2Al2 and Q2Al3).

3.2 Cognitive area and math domain of interest

Difficulties concerning the meaning of variable and of expression depending on a variable are connected to the mathematical domain of algebra and to the cognitive domain of reasoning.

3.3 Educational Aims

By means of the intervention tool, students are helped to grasp the meaning of variable and expression depending on such a variable.
Students are asked to work on the equation of a linear function as to reach specific goals. Students work on the algebraic representation as well as on the graphic representation of the linear function: they can make hypothesis on the formula and check them on the graph, or imagine the graphic solution and modify the algebraic equation so as to reach the desired graph. Students are also explicitly asked to make hypothesis before modifying the formula.

In this intervention tool we put into action specific guidelines of UDL.
Guidelines within Principle 1 (provide multiple means of representation), suggest proposing different options for perception and offering support for decoding mathematical notation and symbols.
Guidelines from Principle 2 (provide multiple means of action and expression) suggest to offer different options for expression and communication supporting planning and strategy development.
Both types of guidelines are followed in the fact that students are provided multiple means of representation (algebraic formulas, graphs) and can dynamically act on them (directly on the algebraic formula, indirectly on the graph, by modifying the formula).
Guidelines from Principle 3 show how certain activities can recruit students' interest, optimizing individual choice and autonomy, and minimizing threats and distractions. Students are involved into a game (catching the stars) that is motivating. Students are encouraged to draw hypothesis and reflect on their choices. Since the activity is made up pf a series of tasks, with increasing difficulty, students are helped to become fluent.

In terms of formative assessment, students can work individually or in small groups and, after each item or at the end of the activity, the teacher can promote a class discussion (formative assessment strategy 2). Students discuss their strategies and difficulties (strategies 4 and 5). The teacher can monitor students' progress throughout the game, giving feedback and prompts (strategy 3).

3.4 Addressing to Student /class

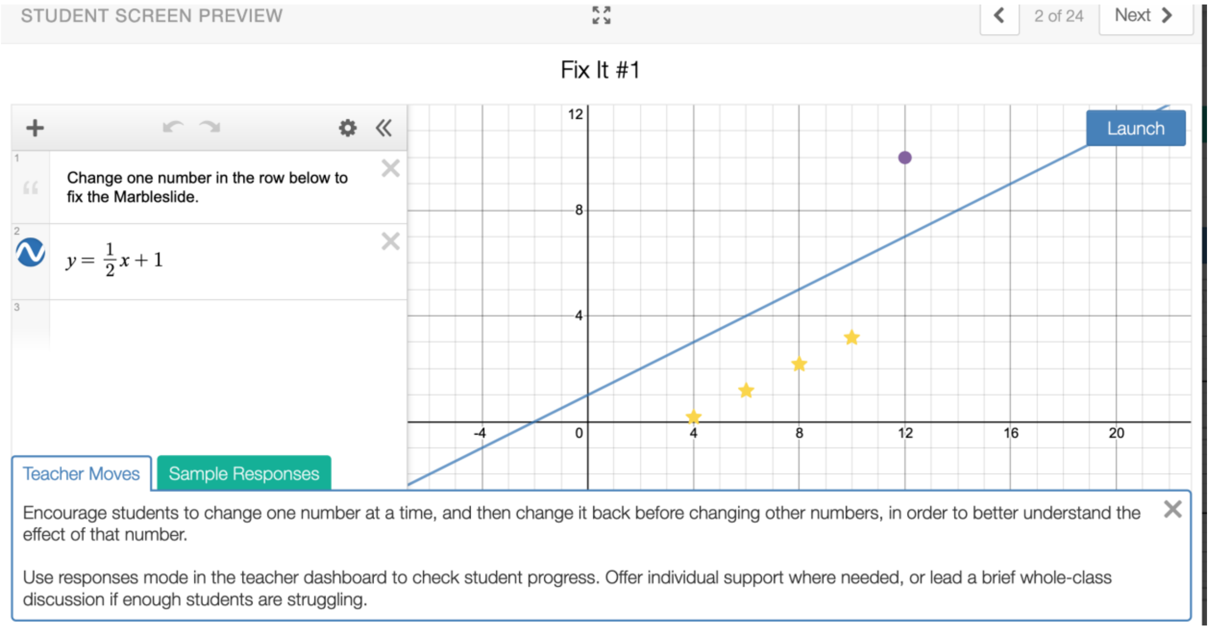
The intervention tool is addressed to all the class. Students can work individually or in small groups.

3.5 Educational activities: the Intervention Tool

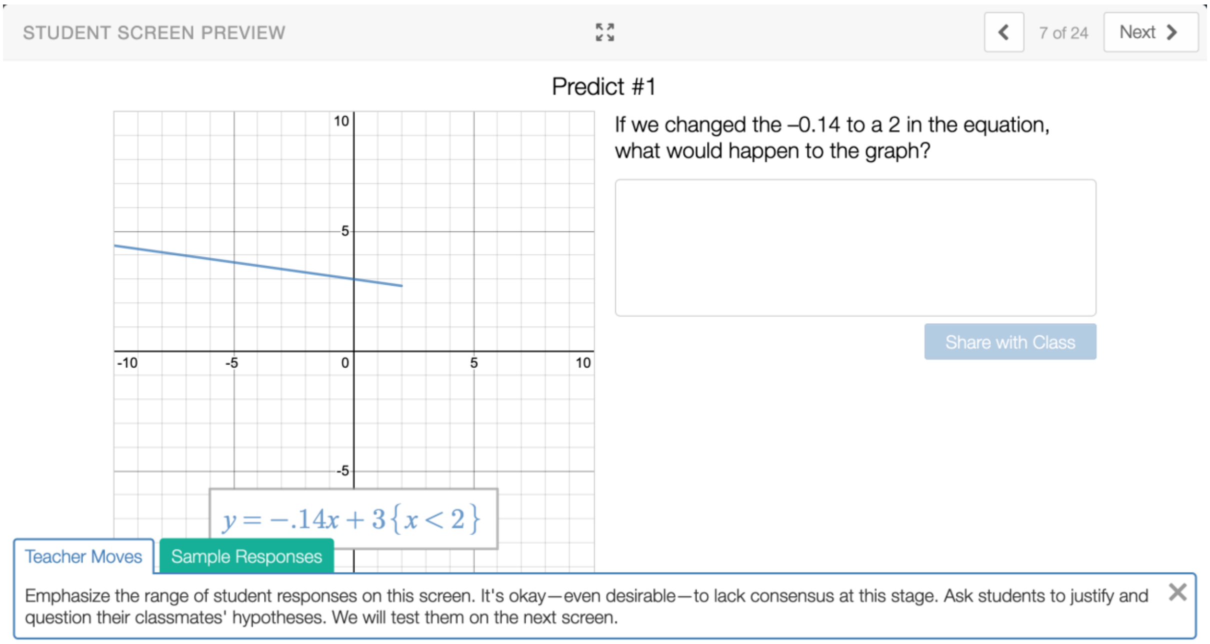
The intervention tool exploits an existing resource, available on the web: https://teacher.desmos.com/activitybuilder/custom/566b31734e38e1e21a10aac8

The resource is developed by means of the dynamic geometry software Desmos and is inserted into the section "Classroom activities".

Students are asked to fill the 24 items of the activity. In each item, they seen on the screen an algebraic window and a geometric one. Students have to modify the algebraic representation of the function so as to obtain a line that guides the "marbles" to go through the stars. Students can test their ideas by launching the marbles, and have a chance to revise before trying the next challenge.



The activity contains also prediction activities, that are followed by a verification step.



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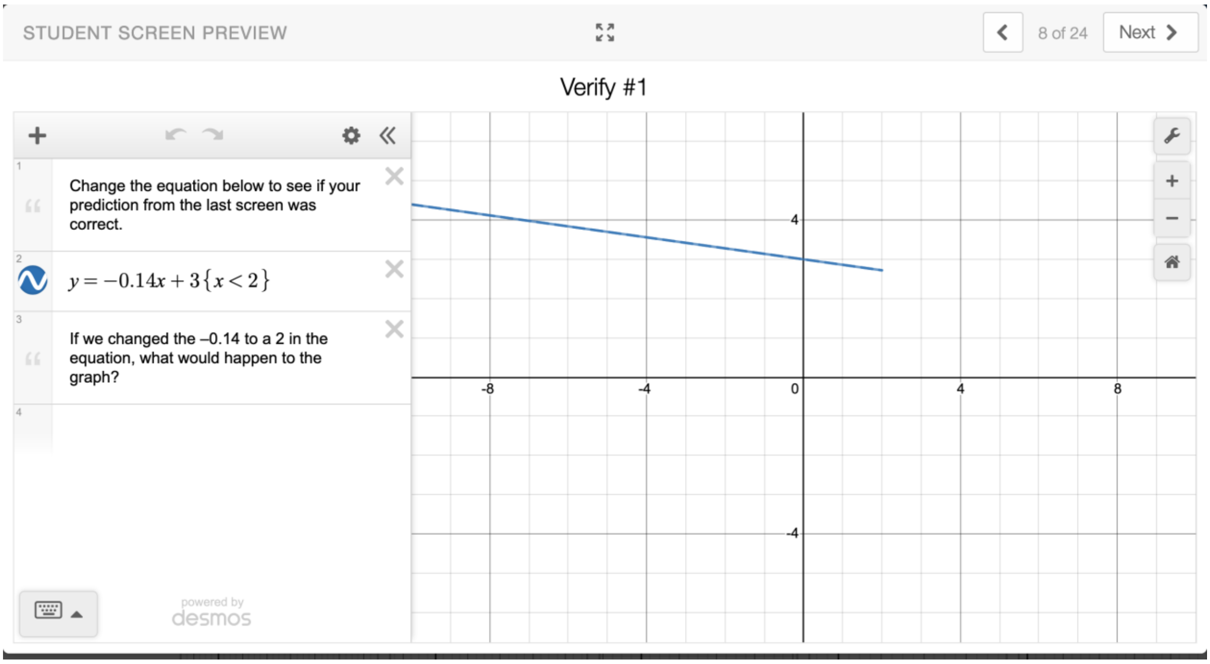

The resource contains student activities as well as a teacher guide. Teachers are suggested to “Encourage students to change one number at a time, and then change it back before changing other numbers, in order to better understand the effect of that number”. Teachers are also suggested to offer individual support where needed, or lead a brief whole-class discussion if many students are struggling.

4. References

References for the theoretical framework are already provided.

For resources, see: https://teacher.desmos.com