Project Number: 2018-1IT02KA201048274

INTERVENTION TOOL

Understanding the meaning of modelling a problem using logical-mathematical skills

1. Introduction

This tool had been created to find general laws through simple understanding to solve problems in algebra.

We are referring to some theoretical frameworks that will be described in session 2.

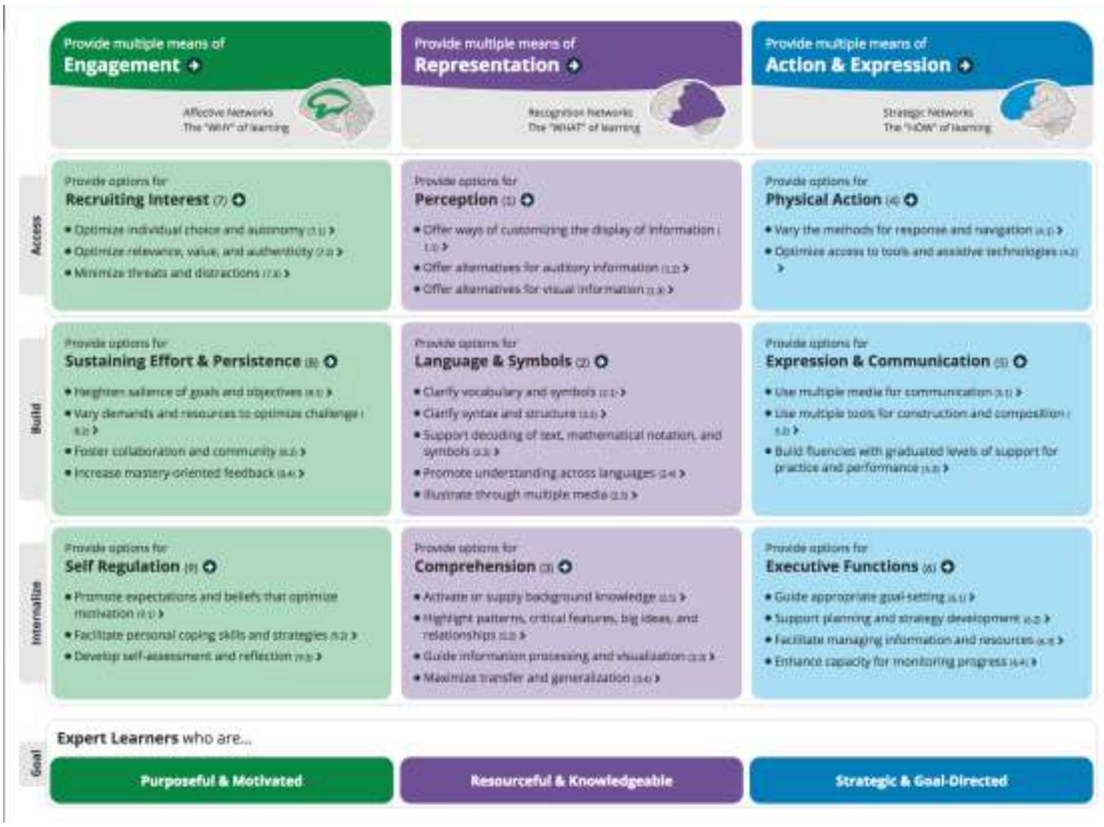
Session 3 describes the design of educational activities. In particular we show the activities proposed to the class, the educational objective of the activities, the cognitive area and the mathematical field of interest and mathematical objects in the areas of difficulty identified through the B2 questionnaire.

2. Theoretical framework

The theoretical references that helped us design the following activities are:

1) Universal Design Principles for Learning (UDL)

The **principles of the UDL** (Table 3),a framework designed specifically to design inclusive educational activities (http://udlguidelines.cast.org/ http://udlguidelines.cast.org/)) are organized in the following table, UDL principles and guidelines



	Provide multiple means of **Engagement** (Affective Networks. The "WHY" of learning)	Provide multiple means of **Representation** (Recognition Networks. The "WHAT" of learning)	Provide multiple means of **Action & Expression** (Strategic Networks. The "HOW" of learning)
Access	Provide options for **Recruiting Interest** (7) • Optimize individual choice and autonomy (7.1) • Optimize relevance, value, and authenticity (7.2) • Minimize threats and distractions (7.3)	Provide options for **Perception** (1) • Offer ways of customizing the display of information (1.1) • Offer alternatives for auditory information (1.2) • Offer alternatives for visual information (1.3)	Provide options for **Physical Action** (4) • Vary the methods for response and navigation (4.1) • Optimize access to tools and assistive technologies (4.2)
Build	Provide options for **Sustaining Effort & Persistence** (8) • Heighten salience of goals and objectives (8.1) • Vary demands and resources to optimize challenge (8.2) • Foster collaboration and community (8.3) • Increase mastery-oriented feedback (8.4)	Provide options for **Language & Symbols** (2) • Clarify vocabulary and symbols (2.1) • Clarify syntax and structure (2.2) • Support decoding of text, mathematical notation, and symbols (2.3) • Promote understanding across languages (2.4) • Illustrate through multiple media (2.5)	Provide options for **Expression & Communication** (5) • Use multiple media for communication (5.1) • Use multiple tools for construction and composition (5.2) • Build fluencies with graduated levels of support for practice and performance (5.3)
Internalize	Provide options for **Self Regulation** (9) • Promote expectations and beliefs that optimize motivation (9.1) • Facilitate personal coping skills and strategies (9.2) • Develop self-assessment and reflection (9.3)	Provide options for **Comprehension** (3) • Activate or supply background knowledge (3.1) • Highlight patterns, critical features, big ideas, and relationships (3.2) • Guide information processing and visualization (3.3) • Maximize transfer and generalization (3.4)	Provide options for **Executive Functions** (6) • Guide appropriate goal-setting (6.1) • Support planning and strategy development (6.2) • Facilitate managing information and resources (6.3) • Enhance capacity for monitoring progress (6.4)
Goal	**Expert Learners** who are... Purposeful & Motivated	Resourceful & Knowledgeable	Strategic & Goal-Directed

Co-funded by the Erasmus+ Programme of the European Union

The European Commission support for the production of this publication does not constitute an endorsement of the contents which reflects the views only of the authors, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

The Centre for Applied Special Technology (CAST) has developed a comprehensive framework around the Concept of Universal Design for Learning (UDL), with the aim of focusing research, development and educational practice on understanding diversity and facilitating learning (Edyburn, 2005). UDL includes a set of principles, articulated in *Guidelines and control points.* The research underpins the UDL's picture is that "students are highly variable in their response to [1] education. [...]" .

Therefore, UDL focus on these individual differences as an important element in understanding and designing effective learning instructions.

For this purpose, UDL advances three fundamental principles: 1) provide more means of representation, 2) provide more means of action and expression, 3) provide multiple means of engagement. In particular, the guidelines within the first principle have to do with the means of perception involved in receiving certain information and "understanding" the information received. On the contrary, the second principle guidelines take into account the processing of information/ideas and their expression. Finally, the third principle guidelines cover the field of "influence" and "motivation", which is also essential in any educational activity.

For this tool will be focused first on representation, including the Perception and Understanding guidelines. The guidelines suggest and proposed different options for perception and offer support to decode perception and understanding. In particular, they propose to the display As for understanding, the guidelines pay attention to activating activate or supply basic knowledge, highlighting patterns, critical characteristics, great ideas and relationships, and visualization and generalization comprehension. Particularly with regard to maximizing transfer and generalization: "All students must be able to generalize and transfer their learning into new contexts. Students vary in the amount of scaffolding they need for memory and transfer in order to improve their ability to access their previous learning."

So, as far as action and expression is concerned, this tool also includes guidelines in "Varying methods for response and navigation" is suitable for the use of material objects. .

In section 4, you'll analyze an example of activity, classifying it based on the type of mathematical learning that was designed and the cognitive area it supports. Vi I will show you how it was designed on the UDL principles in order to make them inclusive and effective for overcoming the mathematical difficulties identified through the B2 questionnaire.

2) Theoretical frameworks for learning mathematical reasoning

“Mathematics is important, but it is also a story. The mathematical reasoning proceeds by formulas and calculations, but even more with the argumentation, with it we lead the pupils to reason and reflect, to confront and question themselves and, through this method, they often learn without even realizing it”

…

“The learning environment must concern the organizational dimension (management of spaces, equipment, times), the didactic dimension, the relational dimension (positive learning climate and shared rules of behavior). At any age, if the students are not accustomed to arguing, the first few times they are asked to do so are perplexed, disoriented, they don't understand the purpose of the request.”

“The teacher must be able to continuously stimulate them to discuss and compare, because it is only by "arguing that you learn to argue" ... ... Arguing in class is also very important to give an opportunity to listen to the arguments of others and take the points weak and strong points of each pupil. Arguing, explaining the why of things, also greatly strengthens the knowledge of the more specific aspects of content, which otherwise are quickly forgotten ...”(Tiziana Bonasso, Universty of Turin, Italy, “Atti del convegno: 8 ottobre 2015” – “Contare e ... raccontare, imparare matematica attraverso il dialogo e il confronto”).

[1] For a complete list of principles, guidelines and checkpoints, and a broader description of CAST activities, visit http://www.udlcenter.org

Co-funded by the Erasmus+ Programme of the European Union

The European Commission support for the production of this publication does not constitute an endorsement of the contents which reflects the views only of the authors, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

Since man has become self-aware, he has always tried to model reality; it's up to us teachers to be able to convey the taste and fun of doing this.
The follow activities are related to daily life, family, class and student group, to developed student reasoning and intuition, to acquire awareness and good rational and logical skills in mathematics, very useful for solving problems.

"By attaching mathematical theory to the real world, in addition to stimulating interest, it promotes active learning, helps to address study as a discovery and promotes the understanding of mathematical concepts. Today it is possible to propose an elementary approach to mathematical modelling since high school, thanks in part to the support of new technologies.
This process allows children to appreciate the potential of mathematical language and provides them with a key to assimilate the theory with awareness. They discover that thanks to mathematical abstraction, the same model is able to represent multiple phenomena even very different from each other. In addition, tools and techniques can be adapted or assembled to handle new issues. A bit like you do with Lego constructions, where few basic elements allow you to realize a wide variety of structures even very complex. "
"The guiding idea of renewal is to develop an experimental attitude towards *Mathematics,* highlighting its key role in the modelling of applied sciences.
The mathematical model of a real-life phenomenon (or problem) is a rationalization process that aims to provide a synthetic and objective description. The phenomenon can thus be examined, possibly controlled and predictions can be made about its evolution. In recent years, the diffusion and emergence of increasingly powerful and sophisticated computing and graphic representation tools has given a strong impetus to the development of mathematical models even in "non-traditional" disciplines." (P. Brandi, R. Ceppitelli, A. Salvadori, University of Perugia, Italy, “Introduzione elementare alla modellizzazione matematica”, 2002)

“Mathematical model allows us to describe the behavior of a wide range of phenomena and systems in nature and society. With simulations, they allow us to predict the evolution of such systems without actually testing them in the real world. This way not only is it cheaper, safer and faster, but it also helps to develop critical and analytical thinking skills in students. After you generate a preliminary model, you can easily consider several scenarios by making minor changes to the model.” (J. A. Conejero)

3. Design
We find lot of difficult in the following exercise of B2 questionnaire:

Q3Ar2. Represent in algebraic form the following game : “Think of a number, double it, add 4, divide by 2, remove the number you thought.”
If you perform the game, you get 2 as result: why?

3.1. Difficulties identified through questionnaire B2
The intervention tool is presented in reference to a specific difficult detected by the questionnaire. The sequence of indications helps to construct the mathematical model of reference, but the result is not immediate, there is always a logical link with the meaning of calculation . It is necessary to make it clear to us the concept of number. Kieran (1996) proposed a model of algebraic activity that served a few years later as the basis for a definition of algebraic thinking in the early grades—a definition that did not hinge on the use of the letter-symbolic (Kieran 2004): “Algebraic thinking in the early grades involves the development of ways of thinking within activities for which the letter-symbolic could be used as a tool, or alternatively within activities that could be engaged in without using the letter-symbolic at all, for example, analyzing relationships among quantities, noticing structure, studying change, generalizing, problem solving, modeling, justifying, proving, and predicting. (p. 149).”

Co-funded by the Erasmus+ Programme of the European Union

SMILD

Project Number: 2018-1IT02KA201048274

3.2. Cognitive area and mathematical domain of interest

The difficulty area identified through the B2 questionnaire is linked to the domain of Arithmetic, reasoning (Table 1)

Table 1: The difficulties detected are related to reasoning of *Arithmetic*

	Arithmetic	**Geometry**	**Algebra**
Memory			
Reasoning	Q3Ar2		
Visual-spatial			

3.3. Educational goals

This intervention tool allows teacher to improve the cognitive area of reasoning in *Arithmetic* from very simple examples that allow you to understand the problem, in some short passage.

3.4. Addressing to Student /class

The intervention tool can be addressed to all the class. The teacher groups students on the base of their different levels of competencies and s/he involves them in the verbalization of the rules of the game

3.5. Educational activities: the Intervention Tool

In this section, the educational activities are described by steps.

1. The teacher gives the students the same pencils, balls or small objects, and asks students to follow the directions in the game using the items available.

a) Think of a number (in this case the number corresponds to the number of balls chosen) for example, a student predict **3** balls: ⊗ ⊗ ⊗

b) Double it : ⊗ ⊗ ⊗ ⊗ ⊗ ⊗

c) Add 4: ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ + ⊗ ⊗ ⊗ ⊗ = ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗

d) Divide by 2: (⊗ ⊗ ⊗ ⊗ ⊗) (⊗ ⊗ ⊗ ⊗ ⊗)

e) Consider only one group

f) Remove the number you thought: ⊗ ⊗ ⊗ ⊗ ⊗ − ⊗ ⊗ ⊗ = ⊗ ⊗

g) The result is: ⊗ ⊗

2. Now the teacher proposes to the students, always divided into groups, to try the game with other initial numbers, using the coloured pencils in the boxes, carefully follow the requests and find out the final result. Giving a resolution time, the group that will finish first will open the discussion to explain the result.

a) Think of a number, for example, students predict **2** balls: ⊗ ⊗

b) Double it: ⊗ ⊗ ⊗ ⊗

c) Add 4: ⊗ ⊗ ⊗ ⊗ + ⊗ ⊗ ⊗ ⊗ = ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗

Co-funded by the Erasmus+ Programme of the European Union

The European Commission support for the production of this publication does not constitute an endorsement of the contents which reflects the views only of the authors, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

d) Divide by 2: (⊗ ⊗ ⊗ ⊗) (⊗ ⊗ ⊗ ⊗)

e) Consider only one

f) Remove the number you thought: ⊗ ⊗ ⊗ ⊗ – ⊗ ⊗ = ⊗ ⊗

g) The result is: ⊗

3. Altogether, teacher and students, find that the result will always, considering also the numbers of the other groups, be the same = ⊗ ⊗

4. The teacher now proposes to change instruction (c) and instead of adding 4, add 6 and to redo the game to find out the result.

a) Think of a number, for example, students predict **3** balls: ⊗ ⊗ ⊗

b) Double it: ⊗ ⊗ ⊗ ⊗ ⊗ ⊗

c) Add 6: ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ + ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ = ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗

d) Divide by 2: (⊗ ⊗ ⊗ ⊗ ⊗ ⊗) (⊗ ⊗ ⊗ ⊗ ⊗ ⊗)

e) Consider only one group

f) Remove the number you thought: ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ – ⊗ ⊗ ⊗ = ⊗ ⊗ ⊗

g) The result is: ⊗ ⊗ ⊗

5. Now the teacher proposes to the students, always divided into groups, to try the game with other initial numbers. The group that will finish first will open the discussion to explain the result. Then a discussion can be opened between the groups to compare the results obtained. The students, working together, find that the end result, with this change, will always be ⊗ ⊗ ⊗

6. Once again the teacher proposes to the students to change the indication (c) and instead off adding 6, add 8, and follow the previous steps from a) to g) and the next discussion point.

7. If no students discover the hiding rules, the teacher shows a path to a common discuss and two things will be discovered:

> 1) The numbers (balls or pencil) to be used in (c), for this game, must be divisible into two equal parts, in fact, by putting a number that cannot be divided by two, the balls (or pencils) taken cannot be separated into two equal groups;
>
> 2) The result does not depend on the number chosen in (a) but in (c) and is exactly equal to its half.

8. In conclusion, after the discussion with the students, the teacher proposes to represent the result obtained with symbols. The teacher divides the blackboard into two equal parts: in one side of blackboard the teacher will write the game step by step as the students did, in the other half of the blackboard, will write the game in mathematical language:

$$\frac{\mathbf{2}\boldsymbol{n}+\boldsymbol{N}}{\mathbf{2}}-\boldsymbol{n}=\frac{\mathbf{2}\boldsymbol{n}}{\mathbf{2}}+\frac{\boldsymbol{N}}{\mathbf{2}}-\boldsymbol{n}=\boldsymbol{n}+\frac{\boldsymbol{N}}{\mathbf{2}}-\boldsymbol{n}=\frac{\boldsymbol{N}}{\mathbf{2}}$$

where: **n**= number thought
N=number added

9. The teacher invites the students to propose the game to friends or family.

4. Discussion through the UDL's guidelines on the above activities
I note that the same educational purpose of 'playing' with numbers is addressed in different ways by acting on the three principles of the UDL (Table 7, in red, *red* my comments to illustrate the link between principles and our activities).

Table 7: Analysis of activities through the UDL Principles Table.

Commitment	***Representation***	***Action and expression***
Recruitment interests	***Perception*** Provide ways to customize the display of information Offering alternatives for hearing information Offer alternatives for visual information Using of small common objects found in the classroom (pencil balls ....)	***Physical action*** Varying response and navigation methods touching the elements to be processed by hand
Supporting effort and persistence	***Language & Simboli*** Simple translation into mathematical language of a discovery	***Expression communication*** discovering as a group that the rule found works
Self-regulatory	***Understanding*** Activate or provide basic knowledge Managing of numbers Highlight patterns, critical features, great ideas and relationships The use of objects (balls, pencils ...) allows to touch the sequence and evolution of the game Information processing guide Maximize transfer and generality through practical tests, done in a group, students can represent the results obtained and predict others	***Executive functions***

Co-funded by the Erasmus+ Programme of the European Union

5. References

[1] P.Brandi R.Ceppitelli A.Salvadori, Introduzione elementare alla modellizzazione matematica, (CD con software applicativo) Università degli Studi di Perugia (2002) http://matematica-old.unibocconi.it/brandi/modelli.htm
[2] J. Alberto Conejero, "Opinion article in the newspaper Levante-EMV on the International Day of Mathematics", Grado Matemáticas - UPV
[3] https://www.mathplanet.com/education/algebra-1/discovering-expressions,-equations-and-functions/composing-expressions
[4] https://www.mathsisfun.com/numbers/index.html
[5] https://www.mathsisfun.com/algebra/index.html
[6] https://www.homeschoolmath.net/teaching/teach-solve-word-problems.php
[7] https://www.mathsisfun.com/place-value.html
[8] https://www.mathsisfun.com/numbers/addition.html
[9] https://www.mat.univie.ac.at/~neum/model.html
[10] https://www.mathsisfun.com/algebra/mathematical-models.html
[11] https://projectfenix1.wordpress.com/
[12] https://difima.i-learn.unito.it/pluginfile.php/8562/mod_resource/content/1/DIFIMA__8_ottobre_Abstract.pdf
[13] https://www.docsity.com/it/appunti-corso-apprendimento-matematico-di-passolunghi/4656437/
[14] C. Kieran, "Algebraic Thinking in the Early Grades: What Is It?", The Mathematics Educator 2004, Vol.8, No.1, 139 - 151