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INTERVENTION TOOL

Understand the meaning of two power properties using logical and manual capabilities

1. Introduction

To be able to find and increase skills in power properties specifically in product and power, we are referring to some theoretical frameworks that will be described in session 2. Session 3 describes the design of educational activities: the activities proposed to the class, the educational objective of the activities, the cognitive area and the mathematical field of interest and mathematical objects in the areas of difficulty identified through the exercise of B2 questionnaire.

2. Theoretical framework

The theoretical references that helped us design the following activities are:

1) Universal Design Principles for Learning (UDL)
The **principles of the UDL** (Table 3), a framework designed specifically to design *inclusive* educational activities (http://udlguidelines.cast.org/) are organized in the following table:

Table: UDL principles and guidelines

	Provide multiple means of **Engagement** Affective Networks The "WHY" of learning	Provide multiple means of **Representation** Recognition Networks The "WHAT" of learning	Provide multiple means of **Action & Expression** Strategic Networks The "HOW" of learning
Access	Provide options for **Recruiting Interest** (7) • Optimize individual choice and autonomy (7.1) • Optimize relevance, value, and authenticity (7.2) • Minimize threats and distractions (7.3)	Provide options for **Perception** (1) • Offer ways of customizing the display of information (1.1) • Offer alternatives for auditory information (1.2) • Offer alternatives for visual information (1.3)	Provide options for **Physical Action** (4) • Vary the methods for response and navigation (4.1) • Optimize access to tools and assistive technologies (4.2)
Build	Provide options for **Sustaining Effort & Persistence** (8) • Heighten salience of goals and objectives (8.1) • Vary demands and resources to optimize challenge (8.2) • Foster collaboration and community (8.3) • Increase mastery-oriented feedback (8.4)	Provide options for **Language & Symbols** (2) • Clarify vocabulary and symbols (2.1) • Clarify syntax and structure (2.2) • Support decoding of text, mathematical notation, and symbols (2.3) • Promote understanding across languages (2.4) • Illustrate through multiple media (2.5)	Provide options for **Expression & Communication** (5) • Use multiple media for communication (5.1) • Use multiple tools for construction and composition (5.2) • Build fluencies with graduated levels of support for practice and performance (5.3)
Internalize	Provide options for **Self Regulation** (9) • Promote expectations and beliefs that optimize motivation (9.1) • Facilitate personal coping skills and strategies (9.2) • Develop self-assessment and reflection (9.3)	Provide options for **Comprehension** (3) • Activate or supply background knowledge (3.1) • Highlight patterns, critical features, big ideas, and relationships (3.2) • Guide information processing and visualization (3.3) • Maximize transfer and generalization (3.4)	Provide options for **Executive Functions** (6) • Guide appropriate goal-setting (6.1) • Support planning and strategy development (6.2) • Facilitate managing information and resources (6.3) • Enhance capacity for monitoring progress (6.4)
Goal	**Expert Learners** who are... **Purposeful & Motivated**	**Resourceful & Knowledgeable**	**Strategic & Goal-Directed**

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The Center for Applied Special Technology (CAST) has developed a comprehensive framework around the Concept of Universal Design for Learning (UDL), with the aim of focusing research, development and educational practice on understanding diversity and facilitating learning (Edyburn, 2005). UDL includes a set of principles, articulated in Guidelines and control points[1].
Therefore, UDL focus on these individual differences as an important element in understanding and designing effective learning instructions.
For this purpose, UDL advances three fundamental principles: 1) provide more means of representation, 2) provide more means of action and expression, 3) provide multiple means of engagement. In particular, the guidelines within the first principle have to do with the means of perception involved in receiving certain information and "understanding" the information received. On the contrary, the second principle guidelines take into account the processing of information/ideas and their expression. Finally, the third principle guidelines cover the field of "influence" and "motivation", which is also essential in any educational activity.
This tool will be focused first on representation, including the perception and understanding guidelines. The guidelines suggest and propose different options for perception and offer support to decode perception and understanding. In particular, the guidelines pay attention in activating or supplying basic knowledge, highlighting patterns, critical features, big ideas and relationship. Particularly with regard to maximizing transfer and generalization: "All students must be able to generalize and transfer their learning into new contexts. Students vary in the amount of scaffolding they need for memory and transfer in order to improve their ability to access their previous learning."
So, as far as action and expression is concerned, this tool also includes guidelines in "Varying methods for response and navigation" is suitable for the use of material objects. .

In section 4, we will analyze an example of activity, classifying it on the type of mathematical learning that was designed for and the cognitive area that are supported. We will show you how it was designed on the UDL principles in order to make them inclusive and effective for overcoming the mathematical difficulties identified through the B2 questionnaire.

2) Theoretical frameworks for learning mathematical logical reasoning

Discussions and reports on this subject can be found everywhere but it is always better to refer to our basic culture, even at the level of methodology, applying the tradition that comes to us from Greek culture.

"The learning of mathematics, such as reading and writing, is the result of a cultural choice, which is the basis of the very idea of schools. It is the development of intelligence through abstraction and symbolic thought: the transition from a world of pure perceptions, from 'common sense' to 'rational discourse', from immersion in appearances and concrete things to abstract entities that allows us to obtain a certain knowledge. This purpose of education is outlined in the works of Plato, who identifies in mathematics the studies that lead on this path." ('Numbers' Ana Maria Millàn Gasca, pag. 105)

Undoubtedly already with Plato's idea of mathematics as 'paideia' (education) we can consider and value the importance of teaching activities.

Plato, in book VII of the Republic, attributes mathematics to a great formative power: numbers, such as geometry ('knowledge of what always is'), force the soul to use intelligence to achieve the truth: "Have you ever observed how those by nature fit for computes are ready

[1] For a complete list of principles, guidelines and checkpoints, and a broader description of CAST activities, visit http://www.udlcenter.org

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and sharp in almost all disciplines; and that the late, if in this discipline are educated they make a progress, and exercised, even if they do not portray any other advantage, however they gain a cume and make progress?"

Plato again:
"That, for family life, for public life and for all kinds of art, no educational discipline has as great an effectiveness as the science of numbers; but the most important thing is that it awakens those who by nature are sleepy and late in intellect and makes them ready to learn, of good memory and insight, making it progress by divine art beyond its natural abilities" (book V).

Fundamental activities are related to daily life family, class and group, to develop reasoning and intuition, to acquire awareness and mental order and to build rational skills useful to improve the skills into power properties in relation to students with specific learning disabilities.

Fundamental activities are related to daily life, family, class and group, to develop reasoning and intuition, to acquire awareness and mental order and to build rational skills useful to face problems also in relation to subjects with specific learning disabilities.
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3. Design

We find some difficulties in the following exercise of B2:

Q4Al1

$a^2 \cdot a^3 = \;......$

$(a^2)^3 = \;......$

3.1 Difficulties identified through questionnaire B2

The intervention tool is presented in reference to a specific difficulty detected by the questionnaire. The powers rule is not immediate, there is always to make a logical connection with the meaning of multiplication, for example. It is necessary for the student to have clear the general concept of number and operations.

3.2 Cognitive area and mathematical domain of interest

The difficulty area identified through the B2 questionnaire is linked to the domain of Algebra and reasoning (Table 1)

Table 1: The difficulties detected are related to the reasoning and cognitive domain of Algebra

	Arithmetic	**Geometry**	**Algebra**
Memory			
Reasoning			Q4Al1
Visuo-space			

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3.3 Educational goals

This intervention tool allows understanding and improving the reasoning in Algebra from very simple examples that allow the students to understand, in some short passages, the properties of the powers.

3.4 Addressing to Student/class

The intervention tool can be addressed to the class. The teacher groups students on the base of their different levels of competencies and s/he involves them in recognizing power properties.

3.5 Educational activities: the Intervention Tool

In this section, the activities are described in detail.

Intervention steps:

1) The teacher reminds the students of the meaning of the power of a number and writes the definition.

The power (or exponent) of a number (or of another element) says how many times to use the number (or element) in a multiplication.
The power is written as a small number to the right and above a number, the base.

$a^1 = a$	(1 time)	→	$\blacksquare^1 = \blacksquare$	(Using boxes as an element of power)
$a^2 = a \cdot a$	(2 times)	→	$\blacksquare^2 = \blacksquare \cdot \blacksquare$	
$a^3 = a \cdot a \cdot a$	(3 times)	→	$\blacksquare^3 = \blacksquare \cdot \blacksquare \cdot \blacksquare$	
$a^4 = a \cdot a \cdot a \cdot a$	(4 times)	→	$\blacksquare^4 = \blacksquare \cdot \blacksquare \cdot \blacksquare \cdot \blacksquare$	
...				

To generalize:

a^n= a^n tells us to multiply **a** by itself, so there are **n** of those **a**'s:

$a^n = a \cdot a \cdot a \cdot \ldots \cdot a$ (n times) → $\blacksquare^n = \blacksquare \cdot \blacksquare \cdot \blacksquare \cdots \cdot \blacksquare$

So, the exponent help us to do not write lots of multiplies.

2) The teacher shows the application of the definition with an example:

$a^2 \cdot a^3 = \cdots$? How can it be solved?

No problem, just apply the definition:

$a^2 \cdot a^3 = a \cdot a \cdot a \cdot a \cdot a = \quad a^5 \quad = \quad a^{(2+3)}$

↙ ↘ ↘ ↗

2times 3times = 5times

In the same way with a generic and real "box symbol" as basis: $\blacksquare^2 \cdot \blacksquare^3 = (\blacksquare \cdot \blacksquare) \cdot (\blacksquare \cdot \blacksquare \cdot \blacksquare) = \blacksquare^5$

In the same way with a number using as a basis" 2" ($a = 2\ or\ \blacksquare = 2$): $(2^2) \cdot (2^3) = (2 \cdot 2) \cdot (2 \cdot 2 \cdot 2) = 2^5 = 32$

↓ ↓
(4) · (8) = 32

In fact applying the property of the powers: $2^2 = 2\cdot 2 = 4$; $2^3 = 2\cdot 2 \cdot 2 = 8$

3) Now the teacher proposes to the groups of students to try to solve some powers with the same method, for example:
$a^1 \cdot a^2 = \cdots$
Student steps to solve $= a \cdot (a \cdot a) = a \cdot a \cdot a = a^3 = a^{(1+2)}$

$a^3 \cdot a^4 = \cdots$
Student steps to solve $= (a \cdot a \cdot a) \cdot (a \cdot a \cdot a \cdot a) = a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a = a^7 = a^{(3+4)}$

4) In this step the teacher, discussing in class, discovers the general law related to the product of powers with the same base:

$$a^m \cdot a^n = a^{(m+n)} \qquad \blacksquare^m \cdot \blacksquare^n = \blacksquare^{(m+n)}$$

With some more examples, as in step 3, this power property could be acquired.

5) Now, the teacher proposes another question to the students:

$(a^2)^3 = \cdots$? How could it be solved?

The teacher applies the power definition, but with a suggestion:
$\mathrm{a}^2 = \mathrm{a} \cdot \mathrm{a} = \blacksquare$

For example, if $\blacksquare = (\circledast \cdot \circledast)$ “in each box there are 2 balls”

So, we can write the question in this way: $(a^2)^3 = (\blacksquare)^3$

Applying the definition of power: $(a^2)^3 = (\blacksquare)^3 = \blacksquare \cdot \blacksquare \cdot \blacksquare$

Using replacement:
$$(a^2)^3 = (\blacksquare)^3 = \blacksquare \cdot \blacksquare \cdot \blacksquare = (\circledast \cdot \circledast) \cdot (\circledast \cdot \circledast) \cdot (\circledast \cdot \circledast)$$

How many balls did we find?

$$\circledast \cdot \circledast \cdot \circledast \cdot \circledast \cdot \circledast \cdot \circledast = a \cdot a \cdot a \cdot a \cdot a \cdot a = a^6$$

↓ ↓ ↗
6 balls 6 times

So you can write: $(a^2)^3 = a^{(2\cdot 3)} = a^6$

6) Now, the teacher proposes to use this rule with a specific base, for example base 2:

$$(2^2)^3 = 2^{(2\cdot 3)} = 2^6 = 64$$

and the teacher discusses with the class that that exercise could be resolve also by:

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$$(2^2)^3 = (4)^3 = 4 \cdot 4 \cdot 4 = 64$$

7) In this step the teacher proposes to the groups of students to try to solve other examples with the same method:

$(a^3)^4 = \cdots$ and considering that: $a^3 = (a \cdot a \cdot a) = \blacksquare$ -> this could be a box

and if in that box we have 3 balls $\blacksquare = \circledast \cdot \circledast \cdot \circledast$ -> we put in the box the number of balls equal to the first exponent.

Appling the rule found, using the definition of power, each group of students have to try to solve:

Solving steps:

$(\blacksquare)^4 = \blacksquare \cdot \blacksquare \cdot \blacksquare \cdot \blacksquare = (\circledast \cdot \circledast \cdot \circledast) \cdot (\circledast \cdot \circledast \cdot \circledast) \cdot (\circledast \cdot \circledast \cdot \circledast) \cdot (\circledast \cdot \circledast \cdot \circledast)$ =(12balls)

$= (a \cdot a \cdot a) \cdot (a \cdot a \cdot a) \cdot (a \cdot a \cdot a) \cdot (a \cdot a \cdot a) = a^{12} = a^{(3 \cdot 4)}$

↓

(12 times)

8)The teacher asks each group of students to use properties of the powers found, using different elements or numbers. Each group has to chose two different elements and two different numbers for a total of four example.

Each group will show an example that the group thinks more representative than others.

9) In the last step, after those example and extensive discussion, the property of the power of powers, can be formalized as a law:

$$(a^m)^n = a^{(m \cdot n)}$$

$$(\blacksquare^m)^n = \blacksquare^{(m \cdot n)}$$

4. Discussion through the UDL's guidelines on the above activities

We note that the same educational purpose of "playing" with the powers is addressed in different ways by acting on the three principles of the UDL (Table 7, in red the comments to illustrate the link between principles and our activities).

Table 7: Analysis of activities through the UDL Principles Table.

Commitment	***Representation***	***Action and expression***
Recruitment interests	***Perception*** Provide ways to customize the display of information Offering alternatives for hearing information Offer alternatives for visual information Use of box symbol and balls to find the properties	***Physical action*** Varying response and navigation methods
Supporting effort and persistence	***Language & Symbols***	***Expression communication*** *In group is easier*

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	With graphic representation	
Self-regulatory	***Understanding*** Activate or provide basic knowledge Highlight patterns, critical features, great ideas and relationships The manual use of the balls to touch the law of exponential power. The laws are presented in specific examples in order to be generalized	***Executive functions***

5. References

[1] Ana Maria Millàn Gasca (2016) ‘Numbers and shapes', p.105 ed. Zanichelli

[2] Plato (book V and VII of ‘The Republic’)

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[8] https://study.com/academy/lesson/what-are-the-five-main-exponent-properties.html

[9]https://www.khanacademy.org/math/in-in-class-7th-math-cbse/x939d838e80cf9307:in-in-7th-powers-exponents/x939d838e80cf9307:in-in-7th-exponents-powers-exponents-properties-1/v/exponent-properties-1

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