Project number: 2018-1-IT02-KA201-048274

INTERVENTION TOOL

Understanding the meaning of mathematical terms

1. Introduction

The purpose of the intervention tool is to better understand mathematical language through a series of activities that include the interpretation of real problems and situations, also through the use of an interactive application. Discussion of problems in classroom with the teacher and between classmates.

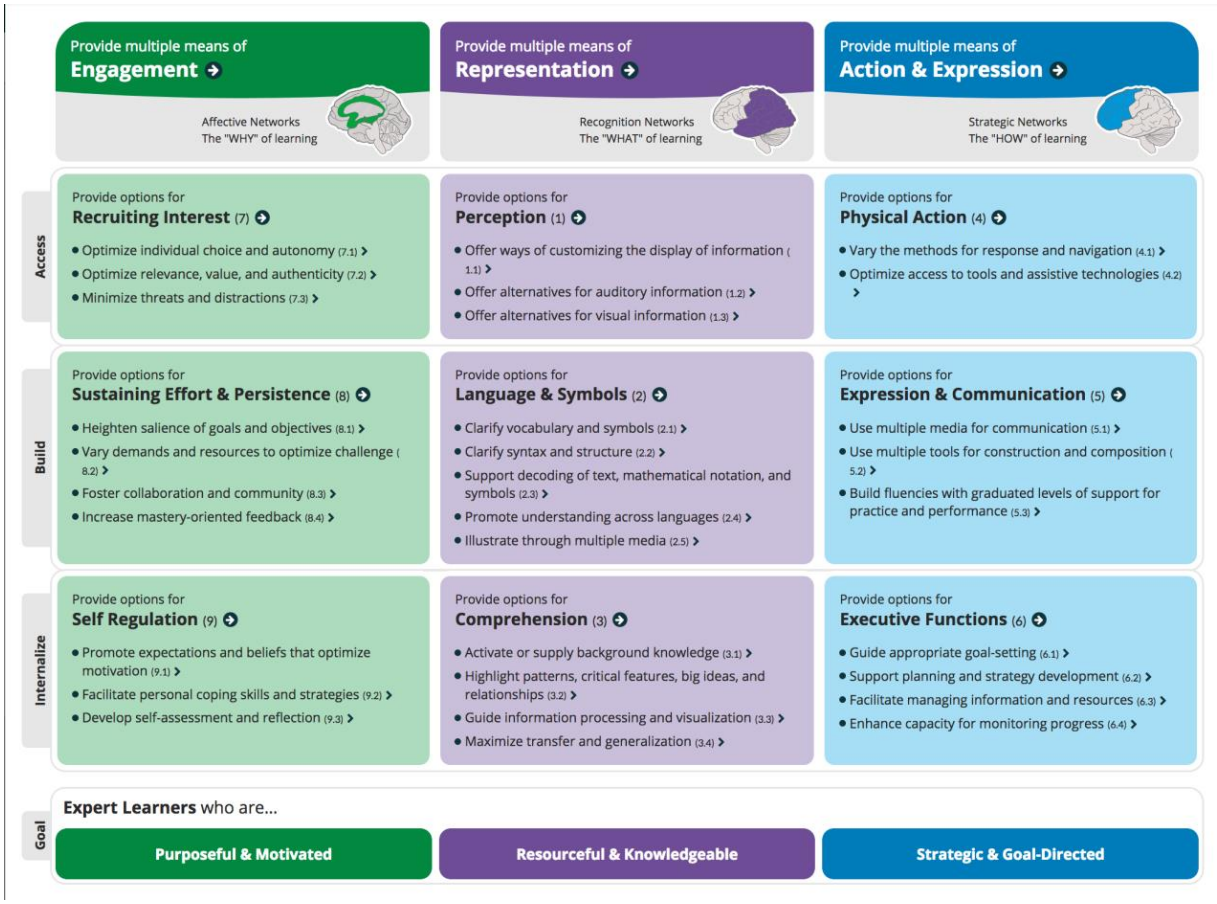
In order to develop a set of educational activities aimed to detect the meaning of arithmetic concepts improving the reasoning skills, we refer to some theoretical frameworks that will be described in the session 2.

In session 3 the design of the educational activities is described. In particular: the activities addressed to the class, the educational aim of the activities, the Cognitive area and math domain of interest and the Mathematical objects in the areas of difficulties identified through the B2 questionnaire.

2. Theoretical framework of reference

The **UDL principles** (Table 3), a framework specifically conceived to design *inclusive* educational activities (http://udlguidelines.cast.org/) are organised in the following table:

Table: UDL principles and guidelines



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The Center for Applied Special Technology (CAST) has developed a comprehensive framework around the concept of Universal Design for Learning (UDL), with the aim of focusing research, development, and educational practice on understanding diversity and facilitating learning (Edyburn, 2005). UDL includes a set of Principles, articulated in *Guidelines and Checkpoints*[1]. The research grounding UDL's framework is that "learners are highly variable in their response to instruction. [...]".
Thus, UDL focus on these individual differences as an important element to understanding and designing effective instruction for learning.

To this aim, UDL advances three foundational Principles:

1) Provide multiple means of representation

2) Provide multiple means of action and expression

3) Provide multiple means of engagement

In particular, guidelines within the first principle have to do with means of perception involved in receiving certain information, and of "comprehension" of the information received. Instead, the guidelines within the second principle take into account the elaboration of information/ideas and their expression. Finally, the guidelines within the third principle deal with the domain of "affect" and "motivation", also essential in any educational activity.

For this tool it will be focused first of all on Representation including the guidelines Perception and Comprehension. The guidelines suggest and propose different options for perception and offer support for decoding perception and comprehension. In particular they propose to offer ways of customizing the display of information. Concerning the comprehension, the guidelines pay attention in activate or supply background knowledge, highlight patterns, critical features, big ideas and relationships, guide information processing and visualisation and maximise transfer and generalisation. In particular regarding to maximise transfer and generalization: "All learners need to be able to generalize and transfer their learning to new contexts. Students vary in the amount of scaffolding they need for memory and transfer in order to improve their ability to access their prior learning".
Then, regarding the Action & Expression this tool includes also the guidelines in "Vary the methods for response and navigation" it is suitable the use of handmade items.

In the section 4 it will analyse an example of activity, classifying it by the type of mathematical learning it is designed and the cognitive area it supports. I will show how this example has been designed on the UDL principles in order to make them inclusive and effective to overcame math difficulties identified through B2 questionnaire. Class discussion is encouraged to better understand the meaning of the terms used. Furthermore, the division into small heterogeneous groups is used to elaborate new problems, which are based on real situations, which will then be shared with the rest of the class. This is to encourage a better understanding of what has been achieved and for greater peer comparison. It is then required to elaborate a personal activity at home as a way of personal reflection and metacognition of what has been learned, which will then be discussed in class with the teacher and the other students. This finds support in Di Martino's article on "Problem solving and mathematical argumentation" which states that "Problem solving and argumentation are key skills in education. Mathematical education at school should make a strong contribution to developing these skills. The promotion of mathematical resolution and argumentation are shared according to numerous international standards. " Furthermore, Mayer states that "An important didactic implication of the focus on metacognition is that problem solving skills should be learned in the context of realistic problem solving situations."

[1] For a complete list of the principles, guidelines and checkpoints and a more extensive description of CAST's activities, visit http://www.udlcenter.org

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3. Design

3.1 Difficulties identified through the B2 questionnaire

The intervention tool is presented in reference to a specific difficulty that was detected by means of the questionnaire.
We detect difficulties in the following items of B2:

Q3Ar1. Solve the following problems:

a) Stella has washed 5 pairs of socks. When she went to take them out of the washing machine one sock was missing. How many socks did Stella take out of the washing machine?
b) Peter has 40 cards. If Alex loses 10 cards, he will have as many as cards Peter does. How many cards does Alex have?
c) One family has 3 children. Each child of the family drinks 2 glasses of milk every day. How many glasses of milk will the family drink during 10 days?
d) To make 4 handbags crocheted in cotton, 6 cotton balls are needed. How many balls do you need to make 20 handbags?
e) Sara received 24 euros as a gift, Marta received 6 euros less. How many euros have the two girls in total?

Q3Ar2. Represent in algebraic form the following game: "Think of a number, double it, add 4, divide by 2, and remove the number you thought"

If you perform the game, you get 2 as a result: why?

The difficulty concerns mathematical reasoning and solving problems relating to arithmetic calculus. Furthermore, in the second question there is a difficulty in translating some mathematical terms such as double, adding, subtracting in mathematical expressions and in the use of variables.

3.2 Cognitive area and math domain of interest

The area of difficulties identified through the B2 questionnaire is related to the domain of Arithmetic and the Reasoning is the cognitive area involved (Table 1).

Table 1: The difficulties detected are linked to the cognitive domain of *Reasoning* in the domain of Arithmetic

	Arithmetic	**Geometry**	**Algebra**
Memory			
Reasoning	**Q3Ar1.** Solve the following problems. **Q3Ar2.** Represent in algebraic form the: "Think of a number, double it, add 4, divide by 2, remove the number you thought" following game, you get 2 as a result: why?		
Visuospatial			

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3.3 Educational Aims

The educational objectives are to improve reasoning skills in the area of arithmetic. In fact, starting from some simple examples in everyday life in which mathematical terms are used, it is then possible to analyze more complex situations in which various mathematical tools are used such as parentheses inside expressions. Then it is possible to generalize the resolution of arithmetic problems using variables.

3.4 Addressing to Student /class

The intervention tool may be addressed to all the class, searching a positive class discussion by students. It is possible to imagine that lot of different cases could arise from the discussion and some new interest could be developed into the students.
In fact, every day we use words such as double, triple and even unwittingly we translate these words into well-defined numbers.

3.5 Educational activities: the Intervention Tool

Here we present a series of educational activities designed for the class. The design of such activities relies on the use of UDL principles in order to make activities inclusive. In particular, we provide multiple means of representation, which promote both students' engagement and their action and expression.

Work of the teacher with the class group:
The teacher interacts with the class building the meaning of some terms and constructs a table in dialogue with the class in which the mathematical operation relating to the term used is specified:

for example:

Term used	**Mathematical operation**	**Example**
Double	Multiply the number by two = $2 \cdot N$ N=number	double 5 = $5 \cdot 2=10$
triple	Multiply the number by three $=3 \cdot N$	triple of 5 = $5 \cdot 3=15$
Half	Divide the number by two=N:2=N/2	half of 6 =6:2=6/2=3
Square	Multiply the number by itself=$N \cdot N=N^2$	The square of 5= $5 \cdot 5=25$
Cube	Multiply the number by itself three times=$N \cdot N \cdot N =N^3$	The cube of 5 =$5 \cdot 5 \cdot 5=125$

The teacher invites students to add more terms and build a more complete table.
The teacher introduces situations in which parentheses must be used. For example: Multiply a number by the sum of the number with its triple and other problems.
Then the teacher presents a real situation, a simple problem, for example: Luca has € 200 he spends half of it buying books and then he spends a quarter of the remaining money buying CDs. If Luca spends another 10 € on a pizza and a drink, how much money will he have left?
The students comment together the meaning of words and express them in mathematical language.
The teacher divides the class into small groups asking that each group formulates at least three problems with examples of real situations and converting them into mathematical language.

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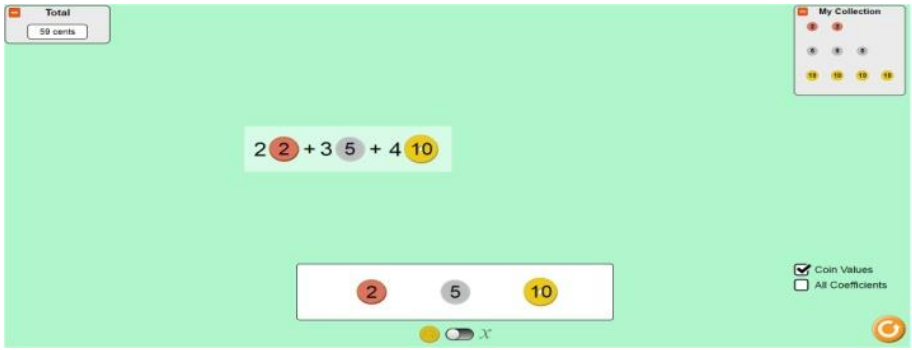
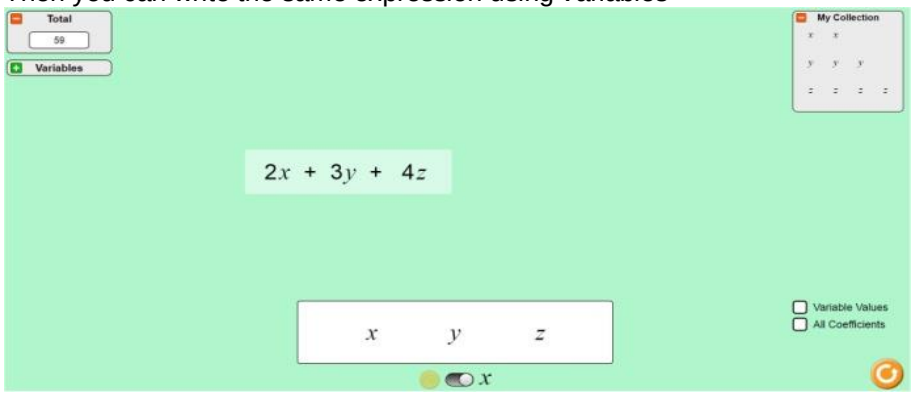
Then there is the discussion of the problems created by the various groups, sharing the meanings and discussing the various cases, highlighting the difficulties encountered. Finally, problems are turned into mathematical expressions.

Subsequently, a work is assigned in which each student must think of at least two problems of situations which will be transformed into mathematical expressions. These problems will be shared with the class and commented.

Construction of an expression through the use of an interactive simulation:
The teacher can use Phet interactive simulations. In particular, the Expression Exchange application can be used. This application is divided into four sections. In the basics section it is possible to build expressions with coins, changing their number and checking their value.



Then you can write the same expression using variables



In this way it is possible to build, through an interactive application, objects reflecting on their value, this mode certainly has a greater utility for MLD students, because they can give objects a visual meaning and a better understanding of the meaning of expressions.
In the game section there are many levels of increasing difficulty in which it is necessary to reproduce the displayed expressions, initially using the coins and then the variables. In addition, a score is awarded when the exercise is well done allowing you to move on to the next level.

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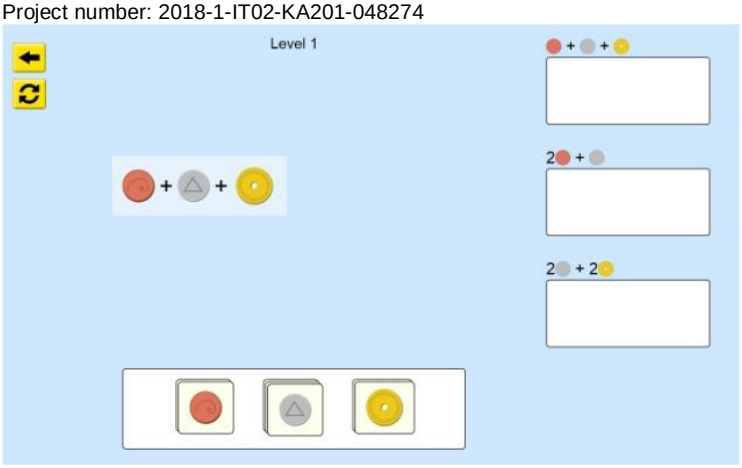

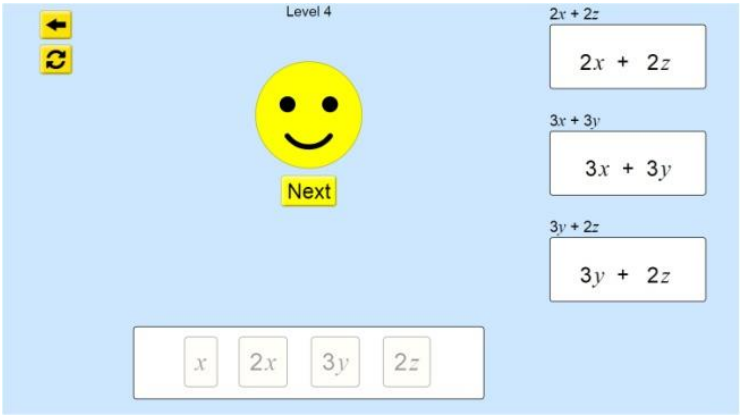

4. Discussion through UDL guidelines about the above-mentioned activities

I observe that the same educational aim of constructing the meaning of "reasoning" in Arithmetic is approached in different ways by acting on the three principles of UDL (Table 7, in *red* my comments to illustrate the connection between the principles and our activities).

Table 7: Analysis of the activities through the Table of UDL principles.

Engagement	***Representation***	***Action & Expression***
Recruiting interest	***Perception*** Offer ways of customizing the display of information Offer alternatives for auditory information Offer alternatives for visual information Different registers through which information are displayed (interactive; visual; symbolic)	***Physical Action*** Vary the methods for response and navigation Optimize access to tools and assistive technologies
Sustaining effort & Persistence Groupwork and class discussion are functional to the aim of	***Language & Symbols*** Clarify vocabulary and symbols Clarify syntax and structure Offer alternative language and symbols to decode information and to work on the information	***Expression Communication*** Groupwork and class discussion may be efficient in "providing differentiated mentors (i.e.,

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fostering collaboration and community. During class discussion the teacher and the peers may provide mastery-oriented feedback	Support decoding of text, mathematical notation, and symbols This is promoted by use a table that specifies the meanings of the main terms and the related mathematical operations.	teachers/tutors who use different approaches to motivate, guide, feedback or inform)", "Provide differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners)".
Self Regulation ***Teacher and peers' feedback may promote subsequent self-regulation.***	***Comprehension*** Activate or supply background knowledge <u>Highlight patterns, critical features, big ideas, and relationships</u> Guide information processing and visualisation Maximise transfer and generalisation	***Executive functions***

5. References

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