Project Number: 2018-1IT02KA201048274

INTERVENTION TOOL

Understanding the relation between fractional numbers and percentages

1. Introduction

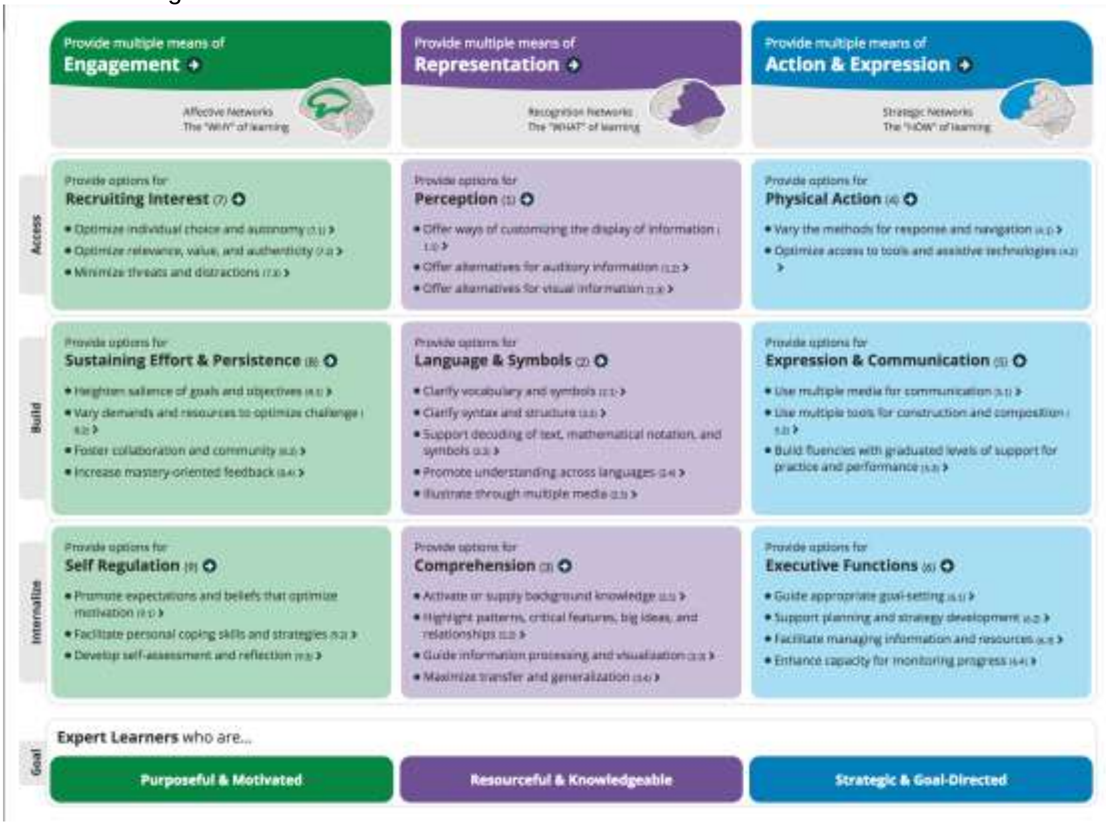
In order to develop a set of educational activities aimed to achieve the relation between fractional numbers and percentages, improving the reasoning skills, we refer to some theoretical frameworks that will be described in the section 2.
In the section 3 the design of educational activities is described in detail. In particular, the activities addressed to the class, the educational aims, the cognitive area and math domain of interest and the Mathematical objects related to the area of difficulties identified through the B2 questionnaire.

2. Theoretical framework of reference

The theoretical references that helped us to design the following activities are:

1) Universal design for learning (UDL) principles (Table 3), a framework specifically conceived to design *inclusive* educational activities (http://udlguidelines.cast.org/).

Table 3: UDL guidelines



The Center for Applied Special Technology (CAST) has developed a comprehensive framework around the concept of Universal Design for Learning (UDL), with the aim of focusing research, development, and educational practice on understanding diversity and facilitating learning (Edyburn, 2005). UDL includes a set of Principles, articulated in

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Guidelines and Checkpoints[1]. The research grounding UDL's framework is that "learners are highly variable in their response to instruction. [...]"
Thus, UDL focus on these individual differences as an important felement to understanding and designing effective instruction for learning.
To this aim, UDL advances three foundational Principles: 1) provide multiple means of representation, 2) provide multiple means of action and expression, 3) provide multiple means of engagement. In particular, guidelines within the first principle have to do with means of perception involved in receiving certain information, and of "comprehension" of the information received. Instead, the guidelines within the second principle take into account the elaboration of information/ideas and their expression. Finally, the guidelines within the third principle deal with the domain of "affect" and "motivation", also essential in any educational activity.
For our analyses we will focus in particular on specific guidelines within the three Principles[2].
Guidelines within Principle 1 (provide multiple means of representation), suggest proposing different options for perception and offering support for decoding mathematical notation and symbols. Moreover, guidelines suggest the importance of providing options for comprehension highlighting patterns, critical features, big ideas, and relationships among mathematical notions. Finally, our analyses will give examples of how software AlNuSet can guide information processing, visualization, and manipulation, in order to maximize transfer and generalization.
Moreover, the guidelines from Principle 2 (provide multiple means of action and expression) suggest to offer different options for expression and communication supporting planning and strategy development. Finally, the guidelines from Principle 3 show how certain activities can recruit students' interest, optimizing individual choice and autonomy, and minimizing threats and distractions.
In the section 4 we will analyze examples of activities, classifying them both by the type of mathematical learning they are designed and the cognitive area they support. We will show how these examples have been designed on the UDL principles in order to make them inclusive and effective to overcame math difficulties identified through B2 questionnaire.

2) The European Project **FasMed,** that focused on formative assessment in mathematics and science, (https://research.ncl.ac.uk/fasmed/).
Formative assessment (FA) is conceived as a method of teaching where "evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited" (Black & Wiliam, 2009, p. 7). FaSMEd project refers to Wiliam and Thompson (2007)'s study, that identifies five key strategies for FA practices in school setting: (a) *clarifying and sharing learning intentions and criteria for success;* (b) *engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding;* (c) *providing feedback that moves learners forward;* (d) *activating students as instructional resources for one another;* (e) *activating students as the owners of their own learning.* The teacher, student's peers and the student him- or herself are the agents that activate these FA strategies.

[1] For a complete list of the principles, guidelines and checkpoints and a more extensive description of CAST's activities, visit http://www.udlcenter.org
[2] The items are taken from the interactive list at http://www.udlcenter.org/research/researchevidence

Table 4: Formative assessment strategies

	Where the learner is going	Where the learner is right now	How to get there
Teacher	**1** Clarifying learning intentions and criteria for success	**2** Engineering effective class-room discussions and other learning tasks that elicit evidence of student understanding	**3** Providing feedback that moves learners forward
Peer	Understanding and sharing learning intentions and criteria for success	**4** Activating students as instructional resources for one another	
Learner	Understanding learning intentions and criteria for success	**5** Activating students as the owners of their own learning	

FaSMEd activities are organized in sequences, that encompass group work on worksheets and class discussion where selected group works are discussed by the whole class, under the orchestration of the teacher. Taking into account formative assessment strategies and technology functionalities, Cusi, Morselli & Sabena (2017, p. 758) designed three types of worksheets for the classroom activity:

> "(1) *problem worksheets:* worksheets introducing a problem and asking one or more questions involving the interpretation or the construction of the representation (verbal, symbolic, graphic, tabular) of the mathematical relation between two variables (e.g. interpreting a time-distance graph);
>
> (2) *helping worksheets*, aimed at supporting students who face difficulties with *the problem worksheets* by making specific suggestions (e.g. guiding questions);
>
> (3) *poll worksheets*: worksheets prompting a poll among proposed options".

The authors identified feedback strategies (Table 5) the teacher may adopt to give feedback to students (Cusi, Morselli & Sabena, 2018, p. 3466). These strategies are employed in the class discussion that is organized by the teacher after the group work on worksheets.

Table 5:

Re-voicing	When the teacher mirrors one student's intervention so as to draw the attention on it. Often, during the revoicing, the teacher stresses with voice intonation some crucial words of the sentence she is mirroring. Rephrasing takes place when the teacher reformulates the intervention of one student, with the double aim of drawing the attention of the class and making the intervention more intelligible to everybody.
Rephrasing	Rephrasing takes place when the teacher reformulates the intervention of one student, with the double aim of drawing the attention of the class and making the intervention more intelligible to everybody. Rephrasing is applied when the teacher feels that the intervention could be useful but needs to be communicated in a better way so as to become a resource for the others. […] The revoicing and rephrasing strategies […] turn one student (the author of the intervention) into a resource for the class.
Rephrasing with scaffolding	When the teacher, besides rephrasing, adds some elements to guide the students' work.

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Relaunching	When the teacher reacts to a student's intervention, which (s)he considers interesting for the class, not giving a direct feedback, but posing a connected question. In this way, by relaunching the teacher provides an implicit feedback […] on the student's intervention, suggesting that the issue is interesting and worth to be deepened or, conversely, has some problematic points and should be reworked on.
Contrasting	Contrasting takes place when the teacher draws the attention on two or more interventions, representing two different positions, so as to promote a comparison. By contrasting, […] the authors of the two positions may be resource for the class as well as responsible of their own learning.

We draw from the FaSMEd experience the idea of creating classroom activities in the formative assessment perspective, which may promote inclusion.

3. Design

3.1 Difficulties identified through the B2 questionnaire

The intervention tool is presented in reference to a specific difficulty that was detected by means of the questionnaire. We detect difficulty in the following item of the B2 Questionnaire (Q3 Ar3):

4/5 of the animals of the farm are cows. Express the number of cows as a percentage of the total of the animals of the farm.

This difficulty, related to the understanding of the relation between fractional numbers and percentages, attests disabilities in reasoning skills.

3.2 Cognitive area and math domain of interest

The area of difficulties identified through the B2 questionnaire is related to the domain of Arithmetic. In particular, the difficulty is related to the understanding of the relation between fractional numbers and percentages. Thus, *Reasoning* is the cognitive area involved (Table 1).

Table 1: The difficulties detected are linked to the cognitive domain of *Reasoning* and in the domain of *arithmetic:*

	Arithmetic	**Geometry**	**Algebra**
Memory			
Reasoning	Q3Ar3: 4/5 of the animals of the farm are cows. Express the number of cows as a percentage of the total of the animals of the farm.		
Visuospatial			

3.3 Educational Aims

The intervention tool is aimed at understanding the relation between fractional numbers and percentages.

3.4 Addressing to Student /class

The Intervention tool is articulated in a set of activities that have to be carried out with all the class, in a perspective of inclusion.

3.5 Educational activities: the Intervention Tool

The teaching sequences are conceived to address specific learning difficulty, within an inclusive perspective. The activities play the role of cognitive training. In particular the student is led to understand the semantic of the whole written task (words problem), the meaning of each specific Math term displayed in the exercise and the step by step procedure to grasp their mathematical relations.

The first idea in designing the activities relais on the proposal of a kind of problem worksheet related to understand the semantic of the exercise. The teacher will write on the dashboard the exercise text, underlining some parts of the sentences as followings:

> *4/5 <u>of the animals of the farm </u>are cows. Express the number of cows as a percentage <u>of the total of the animals of the farm</u>.*

The teacher can promote a discussion among the students about:
"Are the underlined parts of the two sentences referring to the same object?"
"What is this object?"
"Can this object (the total of the animals of the farm) be defined as a whole?"

In addition the teacher can propose a discussion, which supports a conceptualization of fractional numbers as part of a whole, using examples from the real life like the following:

> sharing a pizza with a friend means to manage a whole (a pizza) and a fractional number (the part of the pizza to give to a friend). The pizza can be represented as a circle, divided into four equal parts (figure 1):

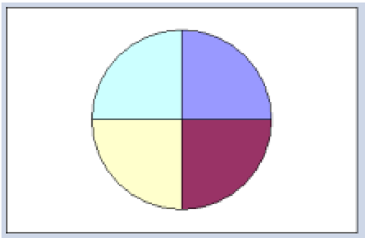

Figure 1

> Each part represents a quarter of the whole pizza or 1/4 of the whole. It could be useful to remind students that a fraction whose numerator is 1 is a unit fraction and it represents 1 shaded part of all the equal parts of the whole. In this example 1/4 is the unit fraction. Similarly, 3/4 of a pizza, means to split a pizza into four slices and take three of these slices, as in the following figure 2:

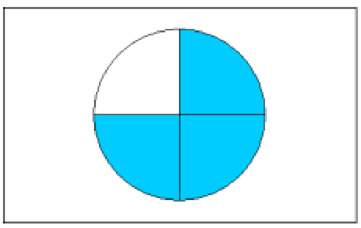

Figure 2

The considered three parts of the whole are equivalent to three times the unit fraction 1/4. The three parts all together represent a new fraction of the whole:

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

As seen in the example in the figure 2, teacher can underline to the students as a **fractional number** can be interpreted as an **operator** dividing the whole into equal parts and considering some of them.

Coming back to the exercise text regarding the animals in the farm, the teacher will ask the students to find the math terms displayed in the task, such as 4/5, a specific fractional number and the word percentage (Latin, *per centum*, meaning per hundred) normally translated into the math code with the symbol "%".

In particular, about fractional numbers, it could be relevant to revise at this step the meaning of E**quivalent Fractional Numbers**. The teacher will guide the discussion both through graphic representations and numbers, so the students will be able to pass from a code to the other (transcoding process). A mathematical model, set in 3 strips, each of the them with as basic criteria, equal length and the same unit of measurement chosen (model scale), will be proposed to the class as is shown in the helping worksheet (figure 3). It's important to focus the students attention on the basic criteria of the model that allow comparing the fractional units represented on each strip:

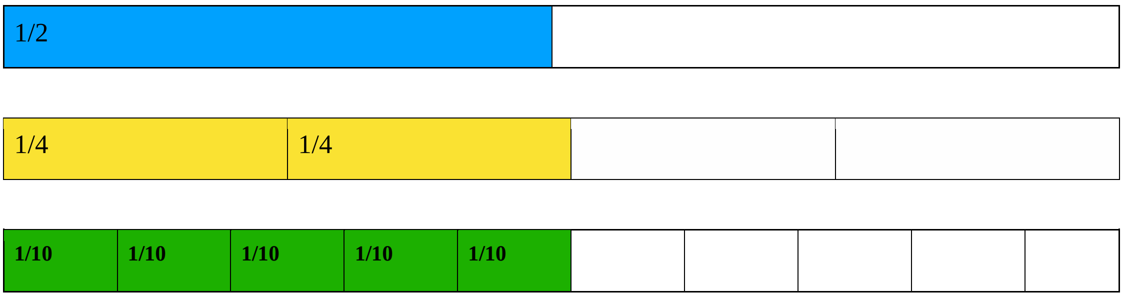

Figure 3 - Revising the meaning of Equivalent Fractional Numbers

Based on the above mentioned criteria, students can see how the fractional numbers displayed in figure 3, 1/2, 2/4 and 5/10 represent the same number 1/2. Asking to the students to add some other examples of equivalent fractions, the students will show to have grasped the meaning of equivalent fractional numbers**.** In addition the teacher will ask the students to formalize the way of obtaining equivalent fractions. A discussion (guided by the teacher) about what students observe in figure 3 and how they can interpret it in arithmetic way, allows students to construct the meaning of fractional numbers as parts of a whole and equivalent fractions.
In terms of formative assessment, strategy 2 is activated. During the discussion, strategies 5 and 4 are activated, since students may intervene to express their doubts (thus becoming owners of their own learning) or to give explanations to their mates (thus becoming resources for the mates).
The teacher and the peers may provide feedback to a student, thus activating strategy 3. A specific attention has to be addressed to the percentage and its meaning. The teacher can give to the students a percentage e.g. 2% asking them to find its equivalent as a fraction having denominator 100, which may be alternatively expressed as "hundredth"/"hundredths".

SMILD

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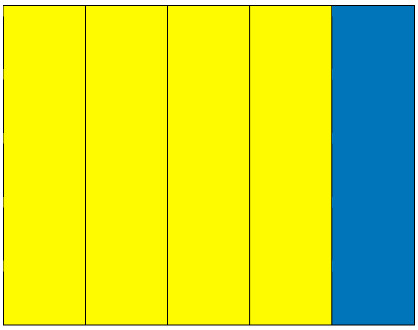
Constructing the relation between fractional numbers and percentages using visual objects.

To allow students to construct the connection between fractional numbers and percentages, the teacher will propose to use a visual representation and ask the students to draw (better on squared paper) a whole, using as a representation a square with length and width equal to 10 times the side of the squares of the paper (figure 4a). Easily the students can interpret the whole equals to 100 paper squares. The squared paper acts as a graphic grid (figure 4b) overlapping and dividing the whole into one hundred equal parts.



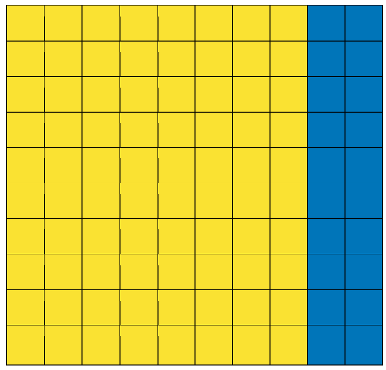
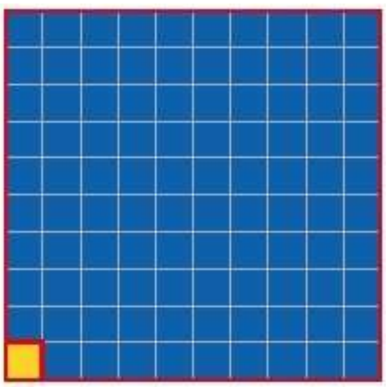


Figure 4a: a square that represents a whole in hundred

Figure 4b: the whole is divided equal parts

Thus, the teacher shows 2 equal squares, each one divided into hundred parts, representing in yellow colour different part of the whole (figure 4c and figure 4d):



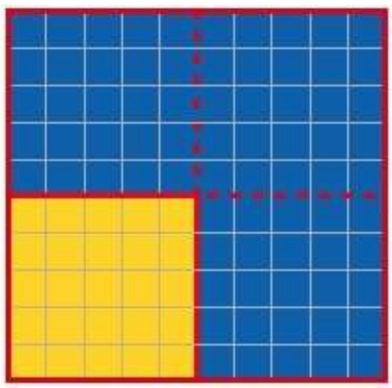


Figure 4c: the yellow quantity= 1 paper square paper

Figure 4d: the yellow quantity= 25 squares

The teacher can highlight in the square referred to figure 4c, one of the 100 parts with yellow colour, promoting a discussion among the students in order to express the yellow quantity of the whole, with the two math codes previously revised (fractions and percentages).

The yellow quantity shown in figure 4c is easily recognized to be equivalent to 1/100 = 1% of the whole. The teacher will submit new examples (e.g. figure 4d) to the class, asking the students to represent new quantities in their squares and translate them into math codes using fractional numbers with denominator 100 and percentages.

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In addition the teacher can launch the discussion asking the following questions: “Which quantity have you drawn in your paper? How do you interpret that quantity using the math codes previously studied?”.
A discussion, guided by the teacher, about what the students observe and how they interpret it, allow the students to construct the relation between fractional numbers, which 100 as denominator, and percentages. In terms of formative assessment, strategy 2 (engineering classroom discussion) is activated. During the discussion, strategy 4 and 5 are also activated, since that the students may intervene to express their doubts (thus becoming owners of their own learning) or to give explanations to their mates (thus becoming resources for the mates). The teacher and the peers may provide feedback to a student, thus activating strategy 3.

Thus the teacher can asks the students to read again the test of Q3Ar3 exercise, posing the following questions: “Due to the given fractional number has denominator different to one hundred, how do you proceed to calculate the equivalent percentage?” and “In comparison with what is shown in the figures 4, which additional step do you have to add to solve the problem?”. A discussion about what students observe on the squares in figure 4 and the meaning of equivalent fractions revised in figure 3, will allow students to construct the connection between numerical fractions and percentages.
The relation between **Fractional Numbers and Percentages**, belonging to arithmetic domain, is constructed in a perceptive way using visual objects. Students will be able to solve the given problem. Thus, as a completion of the task, the teacher will summarize the more effective steps of the implemented problem solving procedure and represent, using visual objects, the problem solution already calculated by the students comparing visual objects (figures 5a and 5b) and math symbols:

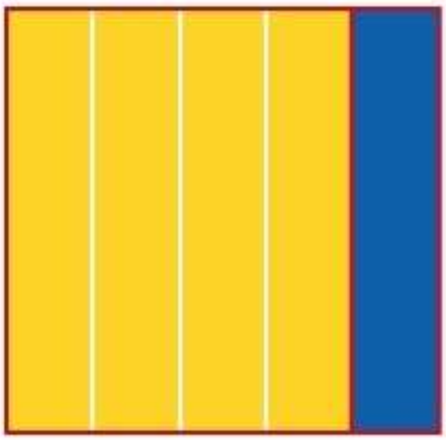

Figure 5a: 4/5 of a whole

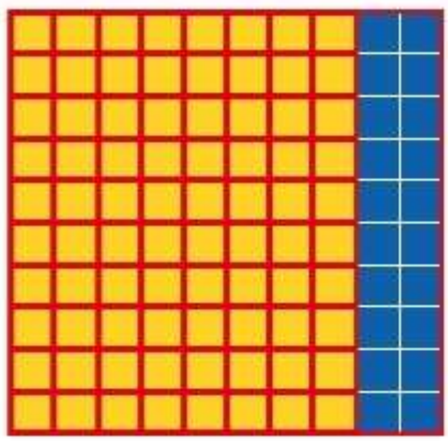

Figure 5b: 80/100 of a whole

In the figure 5a, the yellow part of the square represents 4/5 of a whole as the square has been divided in five equal parts and then four of them have been taken. While 1/5 represents the unit fraction of a whole, the fractional number 4/5 acts as an operator, dividing the whole into equal parts and considering four of them:

4/5 of a whole = (1/5+1/5+1/5+1/5) of a whole

Using the equivalent fractional numbers, 4/5 of a whole can be transformed into a fraction having as denominator 100, applying the following procedure:

$$\frac{4}{5} \text{ of a whole} = \frac{4 \times 20}{5 \times 20} \text{ of a whole} = \frac{80}{100} \text{ of a whole}$$

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As shown in figure 5b, 80/100 of a whole, means to divide a whole in 100 equal parts and take 80 of them. The given task has been solved: 4/5 of the animals of the farm are equivalent to a percentage equals to 80% of them:

$\frac{80}{100}$ **of the animal of the farm = 80 % of the animal of the far**

To allow the students' grasping of the mathematical relations, a series of exercises, focused on the same mathematical content, will be proposed by the teacher (activating strategy 4 and 5). Skills from the memory domain may be elicited as many studies confirm the connection between reasoning domain and memory domain.
The construction of the concept realized thanks to the graphic representation of the squares (visual channel) may allow students, and especially students with MLD, to find mnemonic references that are appropriate for their cognitive style. This allows them to start use representations of these arithmetic concepts at stake, and possibly to place and retrieve them from long term memory in a more effective way.

4. Discussion through UDL guidelines about the above-mentioned activities

We observe that the same educational aim of constructing the connection between numerical fractions and percentages in arithmetic is approached by acting on the three principles of UDL (Table 7, in *red* our comments to illustrate the connection between the principles and our activities).

Table 7: Analysis of the activities through the Table of UDL principles.

Engagement	***Representation***	***Action & Expression***
Recruiting interest	***Perception***	***Physical Action***
Optimize individual choice and autonomy	Offer ways of customizing the display of information	Vary the methods for response and navigation
Optimize relevance, value, and authenticity	Offer alternatives for auditory information	
Minimize threats and distractions	Offer alternatives for visual information	
	Different registers through which information are displayed (symbolic; visual)	

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Sustaining effort Persistence Heighten salience of goals and objectives Vary demands and resources to optimize challenge Foster collaboration and community Increase mastery-oriented feedback Vary demands and resources to optimize challenge Foster collaboration and community Oriented feedbacks support engagement and motivation with respect the elaboration of the solution of the task	***Language & Symbols*** Clarify vocabulary and symbols Clarify syntax and structure Offer alternative language and symbols to decode information and to work on the information Support decoding of text, mathematical notation, and symbols Promote understanding across languages Support decoding of text, math notation and symbols *This is promoted by analyzing the sentences of the exercise from the semantic point of view and proposing different registers at the same time (symbolic and visual)*	***Expression Communication*** Use multiple media for communication Use multiple tools for construction and composition Build fluencies with graduated levels of support for practice and performance To use different registers in order to communicate
Self Regulation Promote expectations and beliefs that optimize motivation Facilitate personal coping skills and strategies Develop self-assessment and reflection *Formative assessment strategies, as discussed in section 2, may help self-assessment and reflection. More specifically, the teacher may provide different types of feedback*	***Comprehension*** Activate or supply background knowledge Highlight patterns, critical features, big ideas, and relationships (checkpoint 3.2) Guide information processing and visualization Maximize transfer and generalization Perception, language and symbols, comprehension (Constructing useable knowledge, knowledge that is accessible for future decision-making, depends not upon merely perceiving information, but upon active "information processing skills")	***Executive functions*** *Guide appropriate goal-setting* *The use of visual objects may also be a support for memory. Visual objects guide students' process of inquiry, providing feedback to their process.* *Support planning and strategy development* *To summarize the more effective steps of the implemented problem solving procedure may be a support to construct the reasoning framework.* Facilitate managing information and resources *Enhance capacity for monitoring progress*

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