Project Number: 2018-1-IT02-KA201-048274

**INTERVENTION TOOL**

# Understanding the Meaning of Volume in solid improving the Visual-Spatial skills

## 1. Introduction

In order to develop a set of educational activities aimed to detect the meaning of volume in solid improving the visual-spatial skills, we refer to some theoretical frameworks that will be described in the session 2.

In session 3 the design of the educational activities is described. In particular: the activities addressed to the class, the educational aim of the activities, the Cognitive area and math domain of interest and the Mathematical objects in the areas of difficulties identified through the B2 questionnaire.

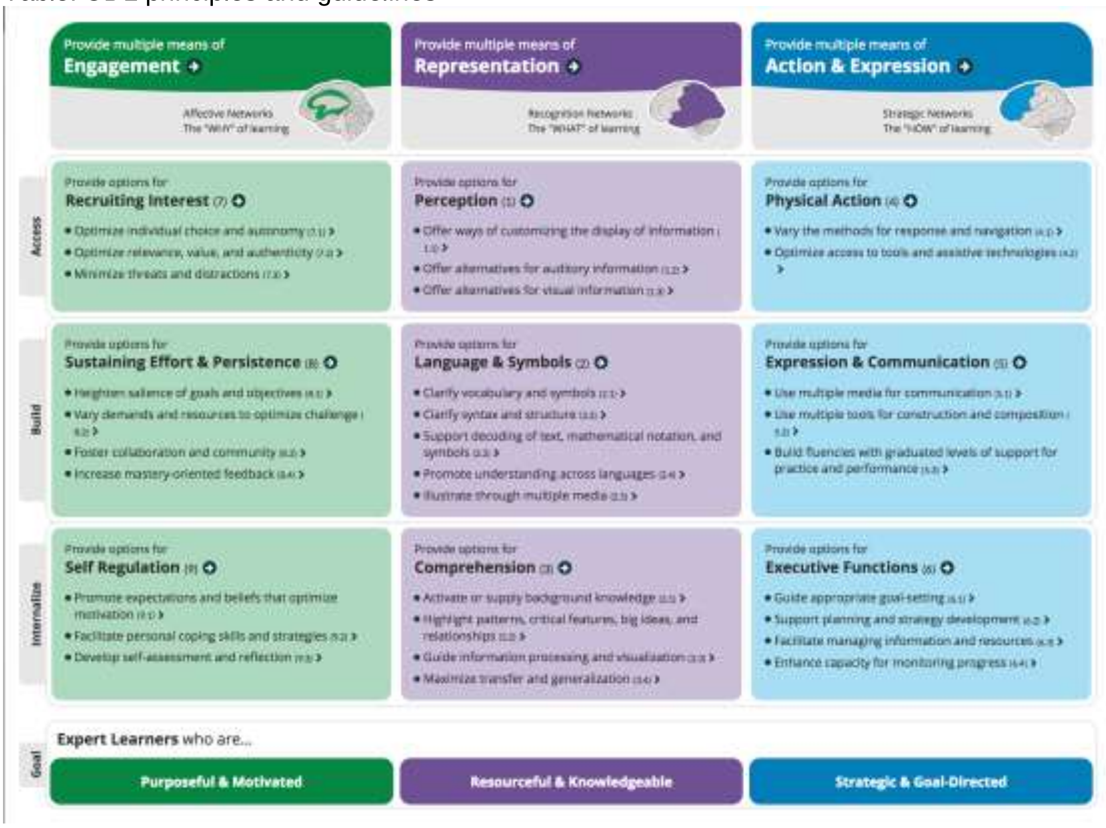
## 2. Theoretical framework of reference

The theoretical references that helped us to design the following activities are:

1) Universal design for learning (UDL) principles

The **UDL principles** (Table 3), a framework specifically conceived to design *inclusive* educational activities ([http://udlguidelines.cast.org/](http://udlguidelines.cast.org/)) are organised in the follow table:

Table: UDL principles and guidelines



| | Provide multiple means of **Engagement** (Affective Networks – The "WHY" of learning) | Provide multiple means of **Representation** (Recognition Networks – The "WHAT" of learning) | Provide multiple means of **Action & Expression** (Strategic Networks – The "HOW" of learning) |
|---|---|---|---|
| Access | Provide options for **Recruiting Interest** (7)<br>• Optimize individual choice and autonomy (7.1)<br>• Optimize relevance, value, and authenticity (7.2)<br>• Minimize threats and distractions (7.3) | Provide options for **Perception** (1)<br>• Offer ways of customizing the display of information (1.1)<br>• Offer alternatives for auditory information (1.2)<br>• Offer alternatives for visual information (1.3) | Provide options for **Physical Action** (4)<br>• Vary the methods for response and navigation (4.1)<br>• Optimize access to tools and assistive technologies (4.2) |
| Build | Provide options for **Sustaining Effort & Persistence** (8)<br>• Heighten salience of goals and objectives (8.1)<br>• Vary demands and resources to optimize challenge (8.2)<br>• Foster collaboration and community (8.3)<br>• Increase mastery-oriented feedback (8.4) | Provide options for **Language & Symbols** (2)<br>• Clarify vocabulary and symbols (2.1)<br>• Clarify syntax and structure (2.2)<br>• Support decoding of text, mathematical notation, and symbols (2.3)<br>• Promote understanding across languages (2.4)<br>• Illustrate through multiple media (2.5) | Provide options for **Expression & Communication** (5)<br>• Use multiple media for communication (5.1)<br>• Use multiple tools for construction and composition (5.2)<br>• Build fluencies with graduated levels of support for practice and performance (5.3) |
| Internalize | Provide options for **Self Regulation** (9)<br>• Promote expectations and beliefs that optimize motivation (9.1)<br>• Facilitate personal coping skills and strategies (9.2)<br>• Develop self-assessment and reflection (9.3) | Provide options for **Comprehension** (3)<br>• Activate or supply background knowledge (3.1)<br>• Highlight patterns, critical features, big ideas, and relationships (3.2)<br>• Guide information processing and visualization (3.3)<br>• Maximize transfer and generalization (3.4) | Provide options for **Executive Functions** (6)<br>• Guide appropriate goal-setting (6.1)<br>• Support planning and strategy development (6.2)<br>• Facilitate managing information and resources (6.3)<br>• Enhance capacity for monitoring progress (6.4) |
| Goal | **Expert Learners** who are... **Purposeful & Motivated** | **Resourceful & Knowledgeable** | **Strategic & Goal-Directed** |

The Center for Applied Special Technology (CAST) has developed a comprehensive framework around the concept of Universal Design for Learning (UDL), with the aim of focusing research, development, and educational practice on understanding diversity and facilitating learning (Edyburn, 2005). UDL includes a set of Principles, articulated in

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*Guidelines and Checkpoints*[1]. The research grounding UDL's framework is that "learners are highly variable in their response to instruction. [...]".

Thus, UDL focus on these individual differences as an important element to understanding and designing effective instruction for learning.

To this aim, UDL advances three foundational Principles: 1) provide multiple means of representation, 2) provide multiple means of action and expression, 3) provide multiple means of engagement. In particular, guidelines within the first principle have to do with means of perception involved in receiving certain information, and of "comprehension" of the information received. Instead, the guidelines within the second principle take into account the elaboration of information/ideas and their expression. Finally, the guidelines within the third principle deal with the domain of "affect" and "motivation", also essential in any educational activity.

For this tool it will be focused first of all on Representation including the guidelines Perception and Comprehension. The guidelines suggest and propose different options for perception and offer support for decoding perception and comprehension. In particular they propose to offer ways of customizing the display of information. Concerning the comprehension, the guidelines pay attention in activate or supply background knowledge, highlight patterns, critical features, big ideas and relationships, guide information processing and visualisation and maximise transfer and generalisation. In particular regarding to maximise transfer and generalization: "All learners need to be able to generalize and transfer their learning to new contexts. Students vary in the amount of scaffolding they need for memory and transfer in order to improve their ability to access their prior learning".

Then, regarding the Action & Expression this tool includes also the guidelines in "Vary the methods for response and navigation" it is suitable the use of handmade items.

In the section 4 it will analyse an example of activity, classifying it by the type of mathematical learning it is designed and the cognitive area it supports. I will show how this example has been designed on the UDL principles in order to make them inclusive and effective to overcome math difficulties identified through B2 questionnaire.

2) Theoretical Frameworks for the Learning of Geometrical Reasoning

From the Geometry Working Group report of the meeting at the King's College, University of London, 28th February 1998 (https://eprints.soton.ac.uk/41308/): "*With the growth in interest in geometrical ideas it is important to be clear about the nature of geometrical reasoning and how it develops. This paper provides an overview of three theoretical frameworks for the learning of geometrical reasoning: the van Hiele model of thinking in geometry, Fischbein's theory of figural concepts, and Duval's cognitive model of geometrical reasoning. Each of these frameworks provides theoretical resources to support research into the development of geometrical reasoning in students and related aspects of visualisation and construction. This overview concludes that much research about the deep process of the development and the learning of visualisation and reasoning is still needed".*

The van Hiele model of thinking in geometry gives the following description of the different levels, based on their translations of the work of van Hiele from the original Dutch:

- Level 0: the student identifies, names, compares and operates on geometric figures;
- Level 1: the student analyses figures in terms of their components and relationships between components and discovers properties/rules empirically;
- Level 2: the student logically inter-relates previously discovered properties/rules by giving or following informal arguments;
- Level 3: the student proves theorems deductively and establishes inter- relationships between networks of theorems;
- Level 4: the student establishes theorems in different postulation systems and analyses / compares these systems.

[1] For a complete list of the principles, guidelines and checkpoints and a more extensive description of CAST's activities, visit http://www.udlcenter.org

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In the theory of figural concepts, Fischbein (1993) observes that while a geometrical figure such as a square can be described as having intrinsically conceptual properties (in that it is controlled by a theory), it is not solely a concept, it is an image too. […] So, Fischbein argues, all geometrical figures represent mental constructs which possess, simultaneously, conceptual and figural properties. […] He argues that geometry is a field in which it is necessary for images and concepts to interact, but that from the student's perspective there can be a tension between the two.
Duval approaches geometry from a cognitive and perceptual viewpoint. In this framework he identifies four types of what he calls "cognitive apprehension":
1. *Perceptual apprehension*: this is what is recognised at first glance; perhaps, for instance, sub-figures which are not necessarily relevant to the construction of the geometrical figure.
2. S*equential apprehension*: this is used when constructing a figure or when describing its construction. In this case, the figural units depend not on perception but on mathematical and technical constraints (in the latter case this could be ruler and compasses, or perhaps the primitives in computer software).
3. *Discursive apprehension*: perceptual recognition depends on discursive statements because mathematical properties represented in a drawing cannot be determined solely through perceptual apprehension, some must first be given through speech.
4. *Operative apprehension*: this involves operating on the figure, either mentally or physically, which can give insight into the solution of a problem.
While the above refers to working with geometric drawings, Duval (1998 p38-39) has gone further in proposing that geometrical reasoning involves three kinds of cognitive processes which fulfil specific epistemological functions. These cognitive processes are:

- Visualisation processes, for example the visual representation of a geometrical statement, the or heuristic exploration of a complex geometrical situation;
- Construction processes (using tools);
- Reasoning processes - particularly discursive processes for the extension of knowledge, for explanation, for proof.

The paper conclusions are: "the above overview of three fairly well-developed frameworks for describing and understanding the development of geometrical reasoning is intended to provide a brief idea of the theoretical resources available which may be useful in research in this area. It also underlines the cognitive complexity of geometry.
As Duval concludes: *much research about the deep process of the development and the learning of visualisation and reasoning are still needed*.

With the idea of these references some very easy tools regarding visual-spatial skills in Geometry are built.

3) Visuospatial abilities and geometry: A first proposal of a theoretical framework for interpreting processes of visualization
The introduction of the paper [2] is:
*We propose a theoretical interpretation of visuo-spatial abilities, as classified in the field of Cognitive Psychology, in the domain of Euclidean Geometry. In this interpretation we make use of Fischbein's theory of figural concepts and of Duval's cognitive apprehensions. Our interpretation lays the foundations for a new theoretical framework that we propose as a tool for qualitative analysis of students' processes of visualization as they carry out geometrical activities. In particular, we present analyses of excerpts from a set of activities designed and proposed in a didactical intervention aimed at strengthening visuo-spatial abilities of a group of students identified as the weakest from a selected 9th grade class of an Italian high school.*
The authors use Fischbein and Duval concepts proposing an activity for students: Imagine a quadrilateral. Focus on the midpoint of each side. Trace the segments that join the midpoints of consecutive sides. What can you tell me about the figure that is formed?

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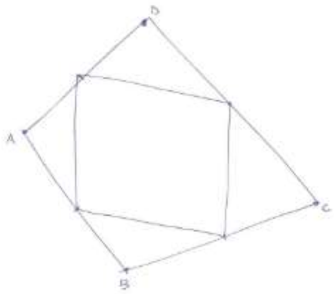


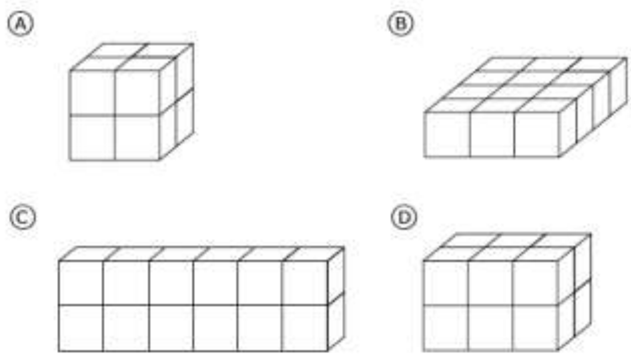

When the student uses his fingers on the desk to draw to better image the figure, the interpretation of the students is that he is using the *imagery manipulation ability*, helping himself with an external image (the quadrilateral with vertexes at his four finger tips) that he can act upon. […] As he moves his fingers (forming what look like various rectangles) he is using geometric prediction, possibly aided by *visual scanning*, to visualize the quadrilateral with vertexes at the midpoints of the sides of the manipulated quadrilateral. […] he never lifts them up from the surface, and then he selects a position which is coherent with respect to the configuration that he wants to (mentally) observe, and starts to move fingers again. The student seems to be able to manipulate the figure in a manner that goes beyond the kind of transformation described by *operative apprehension*. […] The student seems to be looking for extra external support for his *imagery manipulation and geometric prediction abilities*. Moreover, this excerpt is very interesting because of what the student then decides to draw on the sheet of paper when invited to so do. Although he has only mentioned the case in which the quadrilateral is a square and realized with his fingers various cases of it being a rectangle, he draws a much more general convex quadrilateral. This behavior supports their previous hypothesis that the student seems to need external support for his *imagery manipulation and geometric prediction abilities*.
Drawing from these conclusions, we propose here a classroom activity using handmade volumes from a simply white sheet to promote and encourage visual-spatial skills.

### 3. Design

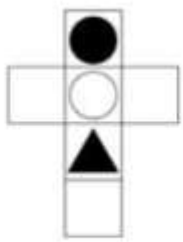
We detect difficulties in the following item of B2:

**Q4G1.**

All the small blocks are the same size. Which stack of blocks has a different volume from the others?



**Q4G2.**



Which of these cubes could be made by folding the figure above?


These difficulties are related to the construction of the meaning of volume in solid and visual-spatial imagery.

### 3.1 Difficulties identified through the B2 questionnaire

The intervention tool is presented in reference to a specific difficulty that was detected by means of the questionnaire. The volume is a measure of how much a figure can hold and it tells us something about the capacity of a figure. The difficulty to visualise the same volumes of some solids built different but with the same quantic solids, i.e. the Q4G1 - B2 questionnaire's exercise need some other deeper studies and the difficult to recognise a solid built starting from the Q4G2 - B2 questionnaire's exercise is a very important disabilities in visual-spatial skills.

### 3.2 Cognitive area and math domain of interest

The area of difficulties identified through the B2 questionnaire is related to the domain of *Geometry* and the *Visual-Spatial* is the cognitive area involved (Table 1).

Table 1: The difficulties detected are linked to the cognitive domain of *Visual-Spatial* and in the domain of *Geometry*

| | Arithmetic | Geometry | Algebra |
|---|---|---|---|
| **Memory** | | | |
| **Reasoning** | | | |
| **Visuo-spatial** | | Q4G1: All the small blocks are the same size. Which stack of blocks has different volume from the others?<br>Q4G2: Which of these cubes could be made by folding the figure above? | |

### 3.3 Educational Aims

This intervention tool permit to investigate and improve the *Visual-Spatial cognitive area* in *Geometry* starting with some very simply figures that permit to understand, in some short passages, the visual-spatial geometry and how these different figures can help with some other more complicated.

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### 3.4 Addressing to Student /class

The intervention tool may be addressed to all the class, searching a positive class discussion by students. It is possible to image that lot of different cases could arise from the discussion and some new interest could be developed into the students. Students meet Geometry all day long, in classroom, at home, everywhere, etc. They could recognise all kinds of solids just by walking and discuss of those in class or at home developing some new "tools" themselves.

### 3.5 Educational activities: the Intervention Tool

In this paragraph the tool activities are described in detail.
The teacher starts at the dashboard to draw a solid, a rectangular prism and asks to students to draw themselves each surface of the solid linked each other, trying to respect the proportion and the initial prism.
After a class discussion the teacher draws the open surface of a different prism, a triangular prism, and then the students must draw the corresponding solid using their sheet and pencil.
A new class discussion should help all students, in particular those who had the difficulties with B2 questionnaire. The teacher will guide, asking what they do (for a sample of the class) and showing those results, good or wrong, focusing the discussion to motivate to do better and to understand the solution in all the students.
Then two different students will create two new exercises, one will start from a solid, the other one will start from open surface. The second one figure, after drawing on the sheet, has to be created as a volume by the students with cutters and tape. A class discussion about what they created and how difficult is, will be opened by teacher.
After these exercises the teachers will ask new exercises at the students who had problems and they could start from solid or open surface as they want.

## 4. Discussion through UDL guidelines about the above-mentioned activities

I observe that the same educational aim of constructing the meaning of "volume" in Geometry is approached in different ways by acting on the three principles of UDL (Table 7, in *red* my comments to illustrate the connection between the principles and our activities).

Table 7: Analysis of the activities through the Table of UDL principles.

| ***Engagement*** | ***Representation*** | ***Action & Expression*** |
|---|---|---|
| ***Recruiting interest*** | ***Perception***<br><br>Offer ways of customizing the display of information<br><br>Offer alternatives for auditory information<br><br>Offer alternatives for visual information<br><br>Many kinds of the same kind of volume are showed and built (visual-dynamic; visual) | ***Physical Action***<br><br>Vary the methods for response and navigation<br><br>*Use of handmade volumes.* |
| ***Sustaining effort & Persistence*** | ***Language & Symbols*** | ***Expression Communication*** |
| ***Self Regulation*** | ***Comprehension***<br><br>Activate or supply background knowledge<br><br>*Using simple volume well know by students at first*<br><br>Highlight patterns, critical features, big ideas, and relationships | ***Executive functions*** |

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| | | |
|---|---|---|
| | *The use of dashboard at first time to create condition to try to image the figures.*<br><br>*Then use the handmade volumes.*<br><br>*At the end using the dashboard to check the visual-spatial skills.*<br><br>Guide information processing and visualisation<br><br>Maximise transfer and generalisation<br><br>*The volumes created are easy to generalise to other figures more complicated or in different situations.* | |

## 5. References


[1] Jones, K. (1998), Theoretical Frameworks for the Learning of Geometrical Reasoning, Proceedings of the British Society for Research into Learning Mathematics, 18(1&2), 29-34.

[2] Elisa Miragliotta, Anna Baccaglini-Frank. Visuo-spatial abilities and geometry: A first proposal of a theoretical framework for interpreting processes of visualization. CERME 10, Feb 2017, Dublin, Ireland. hal-01950545

[3] Baccaglini-Frank, Anna, "Conjecturing in dynamic geometry: A model for conjecture-generation through maintaining dragging" (2010). *Doctoral Dissertations*. 529.

[4] Duval R. (1995) Geometrical Pictures: Kinds of Representation and Specific Processings. In: Sutherland R., Mason J. (eds) Exploiting Mental Imagery with Computers in Mathematics Education. NATO ASI Series (Series F: Computer and Systems Sciences), vol 138. Springer, Berlin, Heidelberg

[5] Karagiannakis, G. N., Baccaglini-Frank, A. E., & Roussos, P. (2016). Detecting strengths and weaknesses in learning mathematics through a model classifying mathematical skills. Australian J. of Learning Difficulties, 21(2), 115–141.



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