Project number: 2018-1-IT02-KA201-048274

INTERVENTION TOOL

Inequalities <, >, ≤, ≥,

1. Introduction

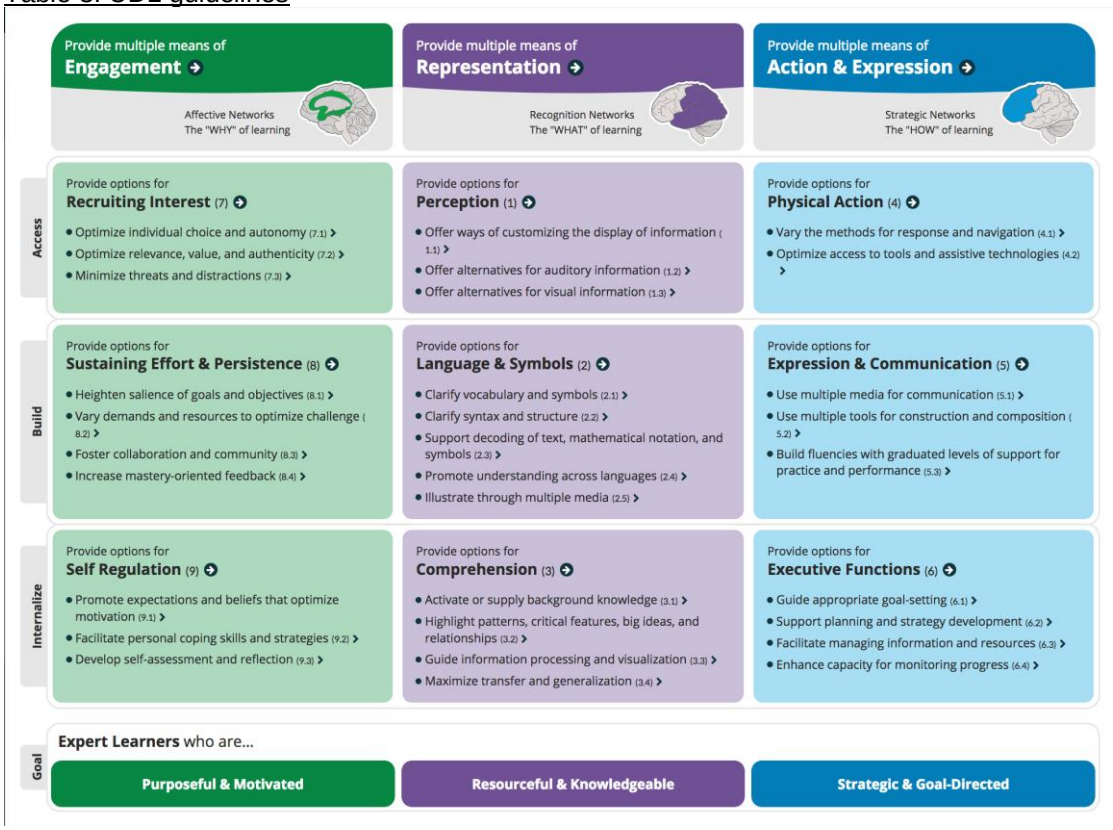
In order to develop a set of educational activities aimed to construct the meaning of variable and expression in a variable, we refer to some theoretical frameworks that will be described in the session
In session 3 the design of the educational activities is described. In particular, if the activities are addressed to a student or the class, the educational aim of the activities, the Cognitive area and math domain of interest and the Mathematical objects in the areas of difficulties identified through the B2 questionnaire

2. Theoretical framework of reference

The theoretical references that helped us to design the following activities are:

1) **Universal design for learning (UDL) principles** (Table 3), a framework specifically conceived to design *inclusive* educational activities (http://udlguidelines.cast.org/)

Table 3: UDL guidelines



The Center for Applied Special Technology (CAST) has developed a comprehensive framework around the concept of Universal Design for Learning (UDL), with the aim of focusing research, development, and educational practice on understanding diversity and facilitating learning (Edyburn, 2005). UDL includes a set of Principles, articulated in *Guidelines and Checkpoints*[1]. The research grounding UDL's framework is that "learners are highly variable in their response to instruction. [...]"

[1] For a complete list of the principles, guidelines and checkpoints and a more extensive description of CAST's activities, visit http://www.udlcenter.org

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Thus, UDL focus on these individual differences as an important felement to understanding and designing effective instruction for learning.
To this aim, UDL advances three foundational Principles: 1) provide multiple means of representation, 2) provide multiple means of action and expression, 3) provide multiple means of engagement. In particular, guidelines within the first principle have to do with means of perception involved in receiving certain information, and of "comprehension" of the information received. Instead, the guidelines within the second principle take into account the elaboration of information/ideas and their expression. Finally, the guidelines within the third principle deal with the domain of "affect" and "motivation", also essential in any educational activity.
For our analyses we will focus in particular on specific guidelines within the three Principles[2].
Guidelines within Principle 1 (provide multiple means of representation), suggest proposing different options for perception and offering support for decoding mathematical notation and symbols. Moreover, guidelines suggest the importance of providing options for comprehension highlighting patterns, critical features, big ideas, and relationships among mathematical notions. Finally, our analyses will give examples of how software AlNuSet can guide information processing, visualization, and manipulation, in order to maximize transfer and generalization.
Moreover, the guidelines from Principle 2 (provide multiple means of action and expression) suggest to offer different options for expression and communication supporting planning and strategy development. Finally, the guidelines from Principle 3 show how certain activities can recruit students' interest, optimizing individual choice and autonomy, and minimizing threats and distractions.
In the section 4 we will analyse examples of activities, classifying them both by the type of mathematical learning they are designed and the cognitive area they support. We will show how these examples have been designed on the UDL principles in order to make them inclusive and effective to overcome math difficulties identified through B2 questionnaire.

2) The European Project **FasMed,** that focused on formative assessment in mathematics and science, (https://research.ncl.ac.uk/fasmed/).
Formative assessment (FA) is conceived as a method of teaching where "evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited" (Black & Wiliam, 2009, p. 7). FaSMEd project refers to Wiliam and Thompson (2007)'s study, that identifies five key strategies for FA practices in school setting: (a) *clarifying and sharing learning intentions and criteria for success;* (b) *engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding;* (c) *providing feedback that moves learners forward;* (d) *activating students as instructional resources for one another;*–(e) *activating students as the owners of their own learning.* The teacher, student's peers and the student him- or herself are the agents that activate these FA strategies.

Table 4: Formative assessment strategies

<table>
<tr><th></th><th>Where the learner is going</th><th>Where the learner is right now</th><th>How to get there</th></tr>
<tr><td>Teacher</td><td rowspan="3">1 Clarifying learning intentions and criteria for success

Understanding and sharing learning intentions and criteria for success

Understanding learning intentions and criteria for success</td><td>2 Engineering effective class-room discussions and other learning tasks that elicit evidence of student understanding</td><td>3 Providing feedback that moves learners forward</td></tr>
<tr><td>Peer</td><td colspan="2">4 Activating students as instructional resources for one another</td></tr>
<tr><td>Learner</td><td colspan="2">5 Activating students as the owners of their own learning</td></tr>
</table>

[2] The items are taken from the interactive list at http://www.udlcenter.org/research/researchevidence

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FaSMEd activities are organized in sequences, that encompass group work on worksheets and class discussion where selected group works are discussed by the whole class, under the orchestration of the teacher. Taking into account formative assessment strategies and technology functionalities, Cusi, Morselli & Sabena (2017, p. 758) designed three types of worksheets for the classroom activity:

> "(1) *problem worksheets:* worksheets introducing a problem and asking one or more questions involving the interpretation or the construction of the representation (verbal, symbolic, graphic, tabular) of the mathematical relation between two variables (e.g. interpreting a time-distance graph);
>
> (2) *helping worksheets*, aimed at supporting students who face difficulties with *the problem worksheets* by making specific suggestions (e.g. guiding questions);
>
> (3) *poll worksheets*: worksheets prompting a poll among proposed options".

The authors identified feedback strategies (Table 5) the teacher may adopt to give feedback to students (Cusi, Morselli & Sabena, 2018, p. 3466). These strategies are employed in the class discussion that is organized by the teacher after the group work on worksheets.

Table 5:

Revoicing	When the teacher mirrors one student's intervention so as to draw the attention on it. Often, during the revoicing, the teacher stresses with voice intonation some crucial words of the sentence she is mirroring. Rephrasing takes place when the teacher reformulates the intervention of one student, with the double aim of drawing the attention of the class and making the intervention more intelligible to everybody.
Rephrasing	Rephrasing takes place when the teacher reformulates the intervention of one student, with the double aim of drawing the attention of the class and making the intervention more intelligible to everybody. Rephrasing is applied when the teacher feels that the intervention could be useful but needs to be communicated in a better way so as to become a resource for the others. […] The revoicing and rephrasing strategies […] turn one student (the author of the intervention) into a resource for the class.
Rephrasing with scaffolding	When the teacher, besides rephrasing, adds some elements to guide the students' work.
Relaunching	When the teacher reacts to a student's intervention, which (s)he considers interesting for the class, not giving a direct feedback, but posing a connected question. In this way, by relaunching the teacher provides an implicit feedback […] on the student's intervention, suggesting that the issue is interesting and worth to be deepened or, conversely, has some problematic points and should be reworked on.
Contrasting	Contrasting takes place when the teacher draws the attention on two or more interventions, representing two different positions, so as to promote a comparison. By contrasting, […] the authors of the two positions may be resource for the class as well as responsible of their own learning.

We draw from the FaSMEd experience the idea of creating classroom activities in the formative assessment perspective, which may promote inclusion.

3. Design

We detect difficulties in the following item of B2:

In the area of core numbers, while ordering the numbers from smallest to greatest, students often have difficulties deciding which numbers are greater or smaller, and being able to solve inequalities;

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3.2. Cognitive area and math domain of interest

The area of difficulties identified through the B2 questionnaire is related to the domain of *Arithmetic*. In particular, the difficulties are related to sort rational numbers on the number line. Thus, *Visuo-Spatial* is the cognitive area involved.

3.3 Educational Aims

The purpose of this activity is to promote understanding which is the number or greater or smaller than a given number or variable.

3.4 Addressing to Student /class

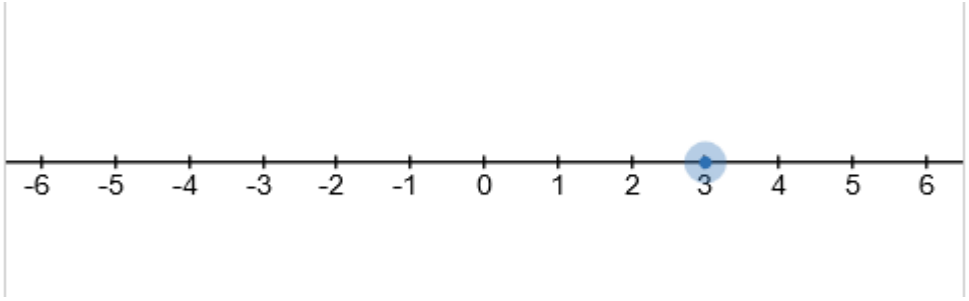
The intervention tool may be addressed to all the class or a group of student students.

3.5 Educational activities: the Intervention Tool

It would be best if students had access to the computers or tablets and could do the activity online using the www.desmos.com website (http://tiny.cc/2bdysz). However, this activity can also be done using paper copies and a whiteboard with markers. The use of interactive tools and software will help to clarify the vocabulary and support decoding of symbols <, >, ≤, ≥ (UDL guidelines).

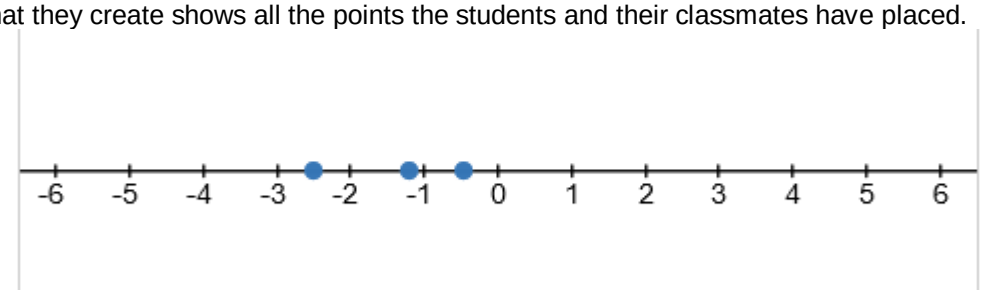
Each student does the following task individually:

1) Drag or draw the blue point to a place on the number line indicating a number that is... ...less than 3.



2) Drag or draw the blue point to ANOTHER place on the number line indicating a number that is... ...less than 3.
3) Drag or draw the blue point to ONE MORE place on the number line indicating a number that is... ...less than 3.

Then, the teacher asks the students to look at the line on the whiteboard and place their points. The graph that they create shows all the points the students and their classmates have placed.



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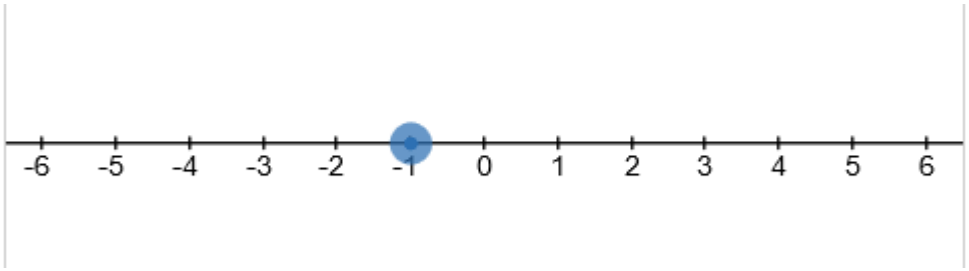
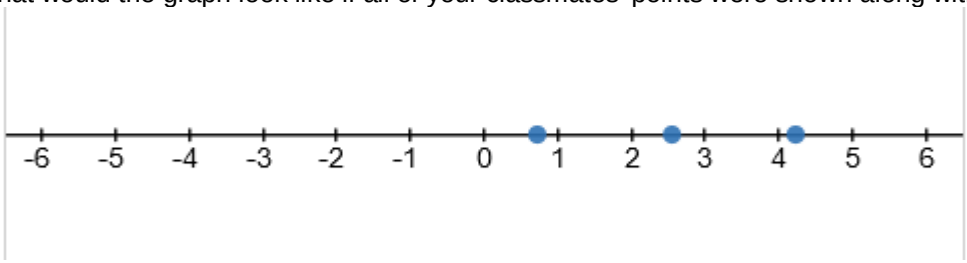
Next, the students do the following task individually.

1) Drag or draw each blue point to a place on the number line indicating a number that is...
...greater than –1.

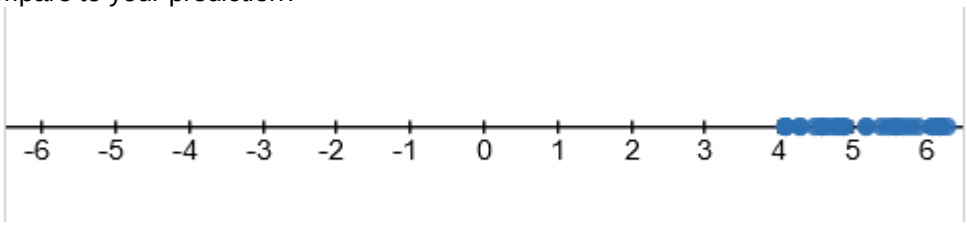


2) What would the graph look like if all of your classmates' points were shown along with yours?



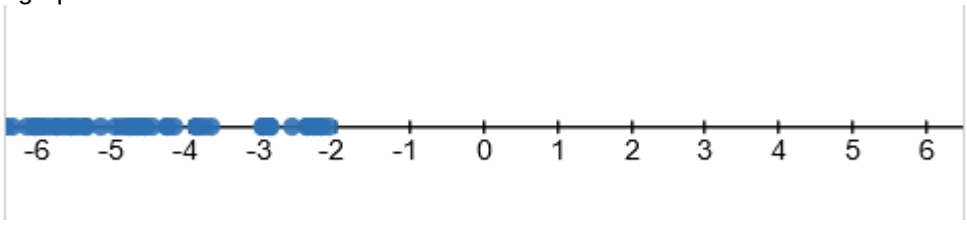
(the graph above presents a hypothetical answer of one student)
The students should notice that the line would be covered in blue dots to the right of -1

Next, the teacher shows the students the following graph and asks the students “How does the actual graph compare to your prediction?”



Students discuss in groups giving their ideas.

Then the teacher gives them the following task.
Here are class results from another challenge. "Drag each point to a place on the number line indicating a number that is..." How would you finish those directions so a group of students would produce a graph similar to this one?



Students identify the numbers on the line and try to come up with the rule. This way they will feel that they are the owners of their learning and they feel more confident.

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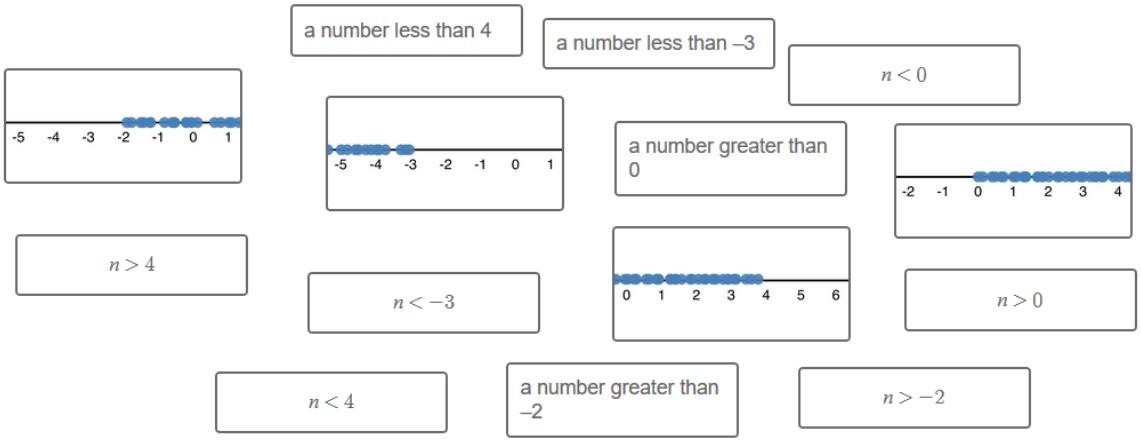
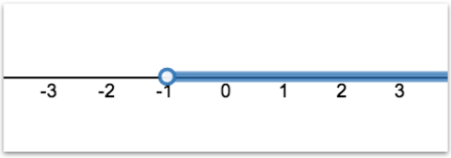
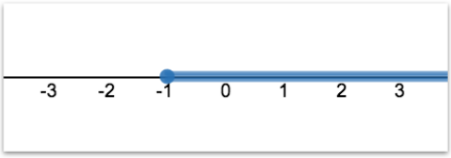
In order to make sure that the students fully grasped the comparison of numbers, they play a matching game. They have to sort the cards into groups of three. (There should be two cards left over.) The teacher explains that the students have to match the graphic representation on the line to the symbol of inequality or an expression.



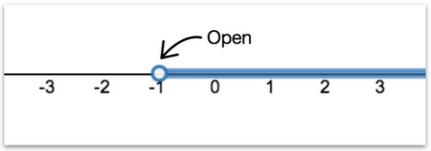
After solving the task, the teacher asks the students if the direction of the arrow in the inequality symbols < > can help them decide which numbers are less than or greater than a given number. Students discuss and therefore deepen their own understanding. They will also share a common vocabulary and graphic representation. (FasMEd)

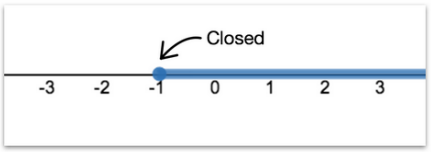
Next task is to make sure the students notice the difference between the symbols
< 'less than' and ≤ 'less than or equal to'
Students look at two number line diagrams. They discuss the following "How are they similar? How are they different?". This will engineer effective classroom discussions and other learning tasks that elicit evidence of student understanding. Students will also come to share a common vocabulary and a common graphic representation. (FasMEd)





The teacher provides them with feedback: "The open dot (above) means that –1 isn't included. The closed dot (below) means that it is. "





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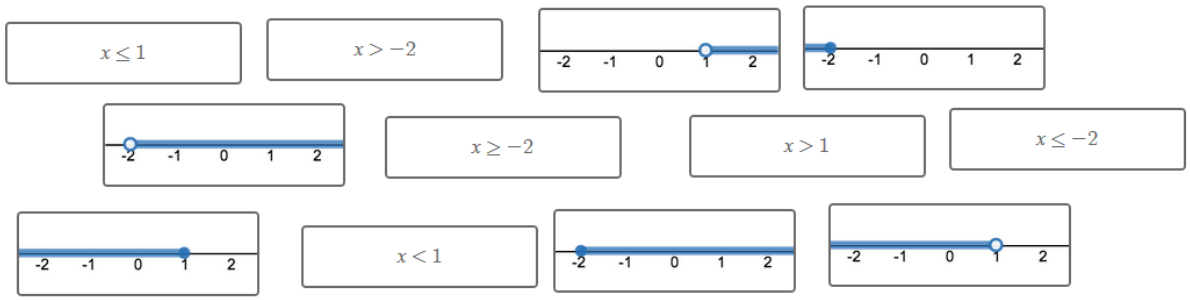
In order to make sure that the students fully grasped the difference between <, >, ≤, ≥, they play a matching game. They have to sort the cards into groups of three. (There should be two cards left over.)

Match each expression with its number line graph.



Finally, in groups or in pair, the students have the analyse the following and answer the question: Which one of these relationships seems different from the rest? In other words, which one doesn't belong?

$x > -2$	$3 < x$
$x \leq 4$	$5 > 6$

The students have a discussion to come up with their own answers. This is the model answer:

- **Top left** (`x>-2`): The only one that includes zero and all positive numbers.
- **Top right** (`3<x`): The only one that has the variable on the right.
- **Bottom left** (`x≤4`): The only one that includes its endpoint.
- **Bottom right** (`5>6`): The only one that is always false.

4. References

[1] Karagiannakis, G. N., Baccaglini-Frank, A. E., & Roussos, P. (2016). Detecting strengths and weaknesses in learning mathematics through a model classifying mathematical skills. Australian J. of Learning Difficulties, 21(2), 115–141. https://doi.org/10.1080/19404158.2017.1289963
[2] Workshop with Dr. Giannis Karagianakis in International
[3] European Project FasMed (https://research.ncl.ac.uk/fasmed/).
[4] Universal design for learning (UDL) principles (http://udlguidelines.cast.org/)

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