Project Number: 2018-1IT02KA201048274

INTERVENTION TOOL

Exploring solutions of equations and systems of equations

1. Introduction

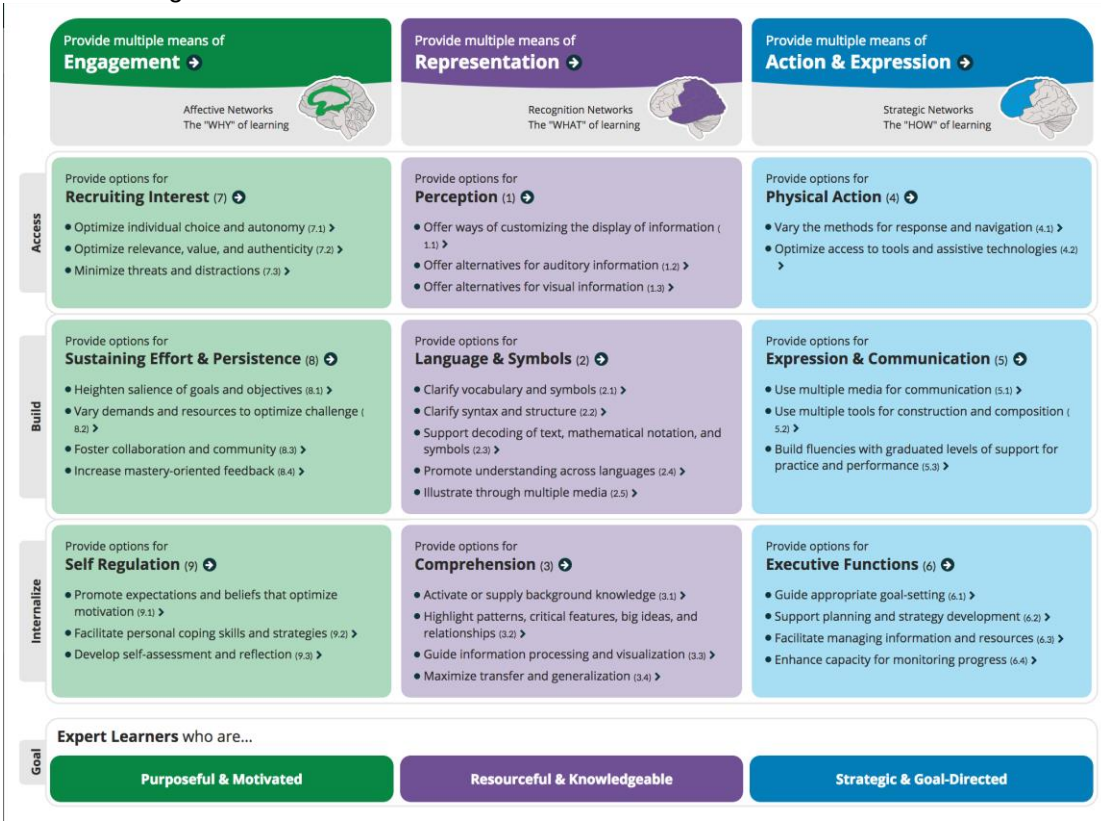
In order to develop a set of educational activities aimed to construction the meaning of solution of an equation and system of equation, we refer to some theoretical frameworks that will be described in the session 2.
In session 3 the design of the educational activities is described. In particular, if the activities are addressed to a student or the class, the educational aim of the activities, the Cognitive area and math domain of interest and the Mathematical objects in the areas of difficulties identified through the B2 questionnaire.

2. Theoretical framework of reference

The theoretical references that helped us to design the following activities are:

1) **Universal design for learning (UDL) principles** (Table 3), a framework specifically conceived to design *inclusive* educational activities (http://udlguidelines.cast.org/)

Table 3: UDL guidelines



The Center for Applied Special Technology (CAST) has developed a comprehensive framework around the concept of Universal Design for Learning (UDL), with the aim of focusing research, development, and educational practice on understanding diversity and facilitating learning (Edyburn, 2005). UDL includes a set of Principles, articulated in

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Guidelines and Checkpoints[1]*.* The research grounding UDL's framework is that "learners are highly variable in their response to instruction. [...]"
Thus, UDL focus on these individual differences as an important filament to understanding and designing effective instruction for learning.
To this aim, UDL advances three foundational Principles: 1) provide multiple means of representation, 2) provide multiple means of action and expression, 3) provide multiple means of engagement. In particular, guidelines within the first principle have to do with means of perception involved in receiving certain information, and of "comprehension" of the information received. Instead, the guidelines within the second principle take into account the elaboration of information/ideas and their expression. Finally, the guidelines within the third principle deal with the domain of "affect" and "motivation", also essential in any educational activity.
For our analyses we will focus in particular on specific guidelines within the three Principles[2].
Guidelines within Principle 1 (provide multiple means of representation), suggest proposing different options for perception and offering support for decoding mathematical notation and symbols. Moreover, guidelines suggest the importance of providing options for comprehension highlighting patterns, critical features, big ideas, and relationships among mathematical notions. Finally, our analyses will give examples of how software AlNuSet can guide information processing, visualization, and manipulation, in order to maximize transfer and generalization.
Moreover, the guidelines from Principle 2 (provide multiple means of action and expression) suggest to offer different options for expression and communication supporting planning and strategy development. Finally, the guidelines from Principle 3 show how certain activities can recruit students' interest, optimizing individual choice and autonomy, and minimizing threats and distractions.
In the section 4 we will analyse examples of activities, classifying them both by the type of mathematical learning they are designed and the cognitive area they support. We will show how these examples have been designed on the UDL principles in order to make them inclusive and effective to overcame math difficulties identified through B2 questionnaire.

2) The European Project **FasMed,** that focused on formative assessment in mathematics and science, (https://research.ncl.ac.uk/fasmed/).
Formative assessment (FA) is conceived as a method of teaching where "evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited" (Black & Wiliam, 2009, p. 7). FaSMEd project refers to Wiliam and Thompson (2007)'s study, that identifies five key strategies for FA practices in school setting: (a) *clarifying and sharing learning intentions and criteria for success;* (b) *engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding;* (c) *providing feedback that moves learners forward;* (d) *activating students as instructional resources for one another;*–(e) *activating students as the owners of their own learning.* The teacher, student's peers and the student him- or herself are the agents that activate these FA strategies.

[1] For a complete list of the principles, guidelines and checkpoints and a more extensive description of CAST's activities, visit http://www.udlcenter.org
[2] The items are taken from the interactive list at http://www.udlcenter.org/research/researchevidence

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Table 4: Formative assessment strategies

	Where the learner is going	Where the learner is right now	How to get there
Teacher	**1** Clarifying learning intentions and criteria for success	**2** Engineering effective class-room discussions and other learning tasks that elicit evidence of student understanding	**3** Providing feedback that moves learners forward
Peer	Understanding and sharing learning intentions and criteria for success	**4** Activating students as instructional resources for one another	
Learner	Understanding learning intentions and criteria for success	**5** Activating students as the owners of their own learning	

FaSMEd activities are organized in sequences, that encompass group work on worksheets and class discussion where selected group works are discussed by the whole class, under the orchestration of the teacher. Taking into account formative assessment strategies and technology functionalities, Cusi, Morselli & Sabena (2017, p. 758) designed three types of worksheets for the classroom activity:

> "(1) *problem worksheets:* worksheets introducing a problem and asking one or more questions involving the interpretation or the construction of the representation (verbal, symbolic, graphic, tabular) of the mathematical relation between two variables (e.g. interpreting a time-distance graph);
>
> (2) *helping worksheets*, aimed at supporting students who face difficulties with *the problem worksheets* by making specific suggestions (e.g. guiding questions);
>
> (3) *poll worksheets*: worksheets prompting a poll among proposed options".

The authors identified feedback strategies (Table 5) the teacher may adopt to give feedback to students (Cusi, Morselli & Sabena, 2018, p. 3466). These strategies are employed in the class discussion that is organized by the teacher after the group work on worksheets.

Table 5:

Revoicing	When the teacher mirrors one student's intervention so as to draw the attention on it. Often, during the revoicing, the teacher stresses with voice intonation some crucial words of the sentence she is mirroring. Rephrasing takes place when the teacher reformulates the intervention of one student, with the double aim of drawing the attention of the class and making the intervention more intelligible to everybody.
Rephrasing	Rephrasing takes place when the teacher reformulates the intervention of one student, with the double aim of drawing the attention of the class and making the intervention more intelligible to everybody. Rephrasing is applied when the teacher feels that the intervention could be useful but needs to be communicated in a better way so as to become a resource for the others. […] The revoicing and rephrasing strategies […] turn one student (the author of the intervention) into a resource for the class.
Rephrasing with scaffolding	When the teacher, besides rephrasing, adds some elements to guide the students' work.

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Relaunching	When the teacher reacts to a student's intervention, which (s)he considers interesting for the class, not giving a direct feedback, but posing a connected question. In this way, by relaunching the teacher provides an implicit feedback […] on the student's intervention, suggesting that the issue is interesting and worth to be deepened or, conversely, has some problematic points and should be reworked on.
Contrasting	Contrasting takes place when the teacher draws the attention on two or more interventions, representing two different positions, so as to promote a comparison. By contrasting, […] the authors of the two positions may be resource for the class as well as responsible of their own learning.

We draw from the FaSMEd experience the idea of creating classroom activities in the formative assessment perspective, which may promote inclusion.

3. Design

3.1 Difficulties identified through the B2 questionnaire

We detect difficulties in the following items of B2:

If a=3 what is the value of 2a+1?

If x= -4, what is the value of 24/x?

a. 6
b. 1/6
c. -1/6
d. -6
e. 20

These difficulties are related to the construction of the meaning of variable and of expression depending on such a variable which will influence the construction of the meaning of equation.

3.2 Cognitive area and math domain of interest

The area of difficulties identified through the B2 questionnaire is related to the domain of *Algebra* and *Reasoning* (Table 1).

Table 1: The difficulties detected are linked to the cognitive domain of *Reasoning* and in the domain of *Algebra*

	Arithmetic	**Geometry**	**Algebra**
Memory			
Reasoning			If a=3 what is the value of 2a+1? If x= -4, what is the value of 24/x? A. 6 B. 1/6 C. -1/6 D. -6 E. 20

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Visuospatial			

3.3 Educational Aims

The intervention tool is aimed at *exploring solutions of equations and systems of equations.*

3.4 Addressing to Student /class

The Intervention tool is articulated in a set of activities that have to be carried out with all the class, in a perspective of inclusion, also in on-line teaching.

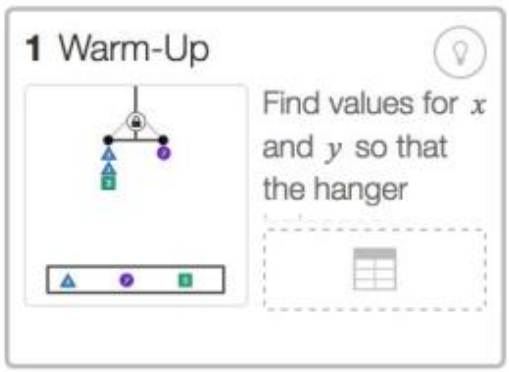
3.5 Educational activities: the Intervention Tool

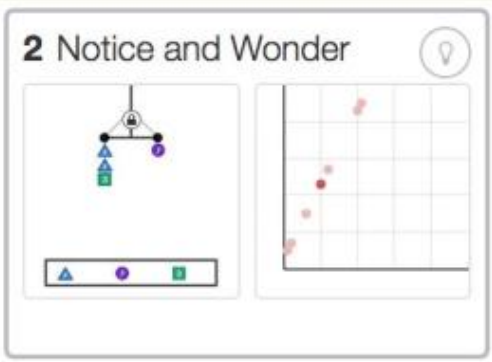
We refer to the activity in Desmos named "Make them balance" (https://teacher.desmos.com).

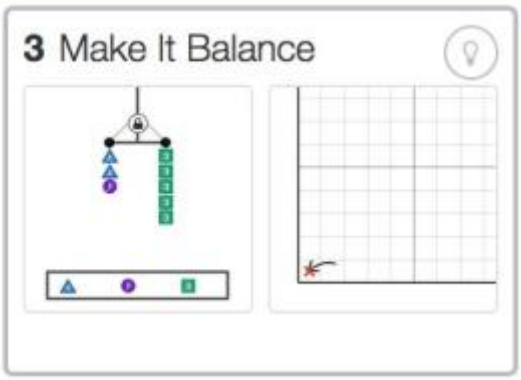
"*This lesson uses hangers to explore solutions to equations and systems of equations. Students first explore a single hanger and see that values that balance the hanger are also solutions to an equation and, when plotted, form a line. Then, students encounter two hangers and see that values that balance both hangers are also solutions to both equations and fall on both lines. Using this context, students make sense of systems of equations that have one solution, no solutions, and infinite solutions*".

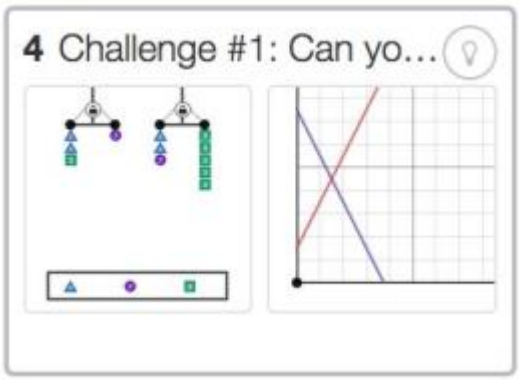
A guideline for teacher is at disposal.

Here the preview of the sequence of activities proposed for the student:









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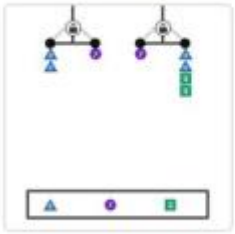
5 System of Equations

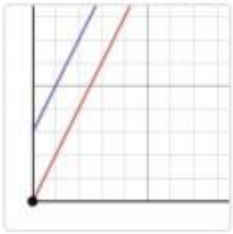
The ordered pair is a solution to the system of

6 Find the Solution

The system of equations below represents the

7 Challenge #2: Can yo…

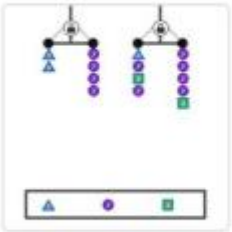


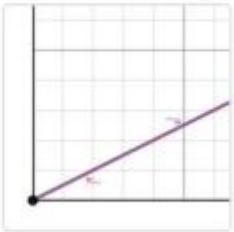


8 No Solutions

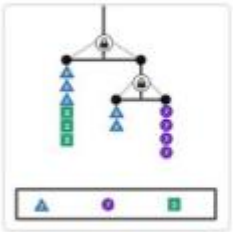
This system of equations from the previous

9 Challenge #3: Can yo…





10 Are You Ready for …



Find values for x and y so that both hangers

11 Reflect

Choose one of the representations

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Note that teacher can interact with each students or planning a session discussion to compare, for instance, different students' solution. This can be done also through on line session.

Discussion through UDL guidelines about the above-mentioned activities

We observe that the same educational aim of constructing the meaning of "variables" , of equation and system of equations in *Algebra* is approached in different ways by acting on the three principles of UDL (Table 7, in *red* our comments to illustrate the connection between the principles and our activities).

Table 7: Analysis of the activities through the Table of UDL principles.

Engagement	***Representation***	***Action & Expression***
Recruiting interest **Optimize individual choice and autonomy** **Optimize relevance, value, and authenticity** **Minimize threats and distractions**	***Perception*** Offer ways of customizing the display of information Offer alternatives for visual information Different registers through which information are displayed (visual; symbolic)	***Physical Action*** Vary the methods for response and navigation
Sustaining effort Persistence **Heighten salience of goals and objectives** **Increase mastery-oriented feedback** **Vary demands and resources to optimize challenge** **Foster collaboration and community** **Oriented feedbacks support engagement and motivation with respect the elaboration of the solution of the task**	***Language & Symbols*** Clarify vocabulary and symbols Clarify syntax and structure Offer alternative language and symbols to decode information and to work on the information Support decoding of text, mathematical notation, and symbols *This is promoted by the dynamic action, and by the manipulation of objects* Promote understanding across languages	***Expression Communication*** Use multiple tools for construction and composition Build fluencies with graduated levels of support for practice and performance To use different registers in order to communicate *This is promoted by the use of terms that are alternative to the formal ones to speak about mathematical objects. Moreover, in the activities virtual or concrete mathematical manipulatives are provided. For instance, dragging a moving point may help visualizing that the variable may have different values on the number line.*

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Self Regulation	***Comprehension***	***Executive functions***
Promote expectations and beliefs that optimize motivation **Facilitate personal coping skills and strategies** **Develop self-assessment and reflection** ***Formative assessment strategies, as discussed in section 2, may help self-assessment and reflection. More specifically, the teacher may provide different types of feedback***	Activate or supply background knowledge Highlight patterns, critical features, big ideas, and relationships (checkpoint 3.2) Guide information processing and visualization Maximize transfer and generalization Perception, language and symbols, comprehension (Constructing useable knowledge, knowledge that is accessible for future decision-making, depends not upon merely perceiving information, but upon active "information processing skills")	*Guide appropriate goal-setting* *The use of artefacts may also be a support for memory. Artefacts guide students' process of inquiry, providing feedback to their process.* *Support planning and strategy development* *Facilitate managing information and resources* *Enhance capacity for monitoring progress*

This allows students to construct meaning for the arithmetic notions at stake.

5. References

[1]https://www.desmos.com/?lang=it

[2]Karagiannakis, G. N., Baccaglini-Frank, A. E., & Roussos, P. (2016). Detecting strengths and weaknesses in learning mathematics through a model classifying mathematical skills. Australian J. of Learning Difficulties, 21(2), 115–141.

[3]UDL Principles: http://udlguidelines.cast.org/

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