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INTERVENTION TOOL

Supporting Memory in Remember Theorem

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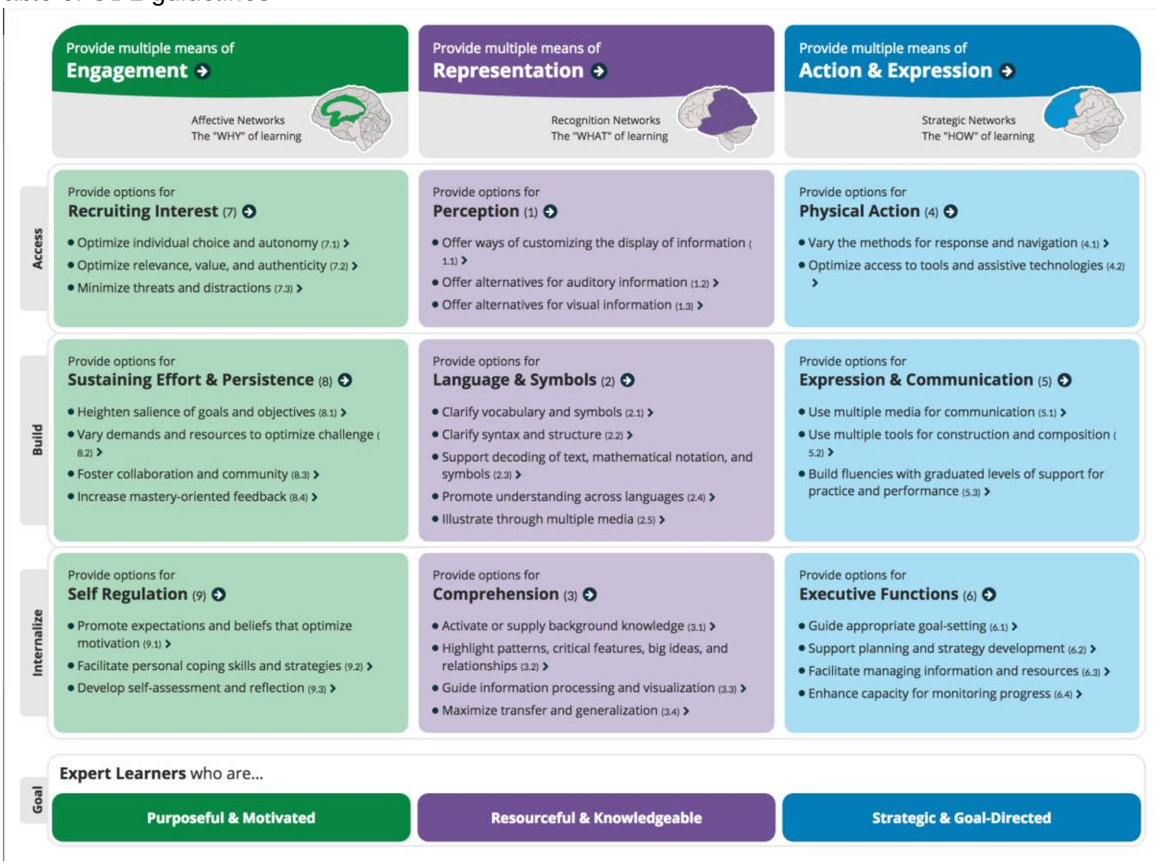
1. Introduction

In order to develop educational activities aimed to support memory in geometry, we refer to some theoretical frameworks that will be described in the session 2.
In session 3 the design of the educational activities is described. In particular, if the activities are addressed to students or the class, the educational aim of the activities, the Cognitive area and math domain of interest and the Mathematical objects in the areas of difficulties identified through the B2 questionnaire

2. Theoretical framework of reference

The theoretical references that helped us to design the following activities are:
1) Universal design for learning (UDL) principles (Table 3), a framework specifically conceived to design inclusive educational activities (http://udlguidelines.cast.org/)

Table 3: UDL guidelines



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The Center for Applied Special Technology (CAST) has developed a comprehensive framework around the concept of Universal Design for Learning (UDL), with the aim of focusing research, development, and educational practice on understanding diversity and facilitating learning (Edyburn, 2005). UDL includes a set of Principles, articulated in Guidelines and Checkpoints2. The research grounding UDL's framework is that "learners are highly variable in their response to instruction. [...]"
Thus, UDL focus on these individual differences as an important element to understanding and designing effective instruction for learning.
To this aim, UDL advances three foundational Principles: 1) provide multiple means of representation, 2) provide multiple means of action and expression, 3) provide multiple means of engagement. In particular, guidelines within the first principle have to do with means of perception involved in receiving certain information, and of "comprehension" of the information received. Instead, the guidelines within the second principle take into account the elaboration of information/ideas and their expression. Finally, the guidelines within the third principle deal with the domain of "affect" and "motivation", also essential in any educational activity.
For our analyses we will focus in particular on specific guidelines within the three Principles3.
In order to characterize students' difficulties in geometry, we refer to the following elements of Karagiannakis' and colleagues' frame (Table 1), which dealt with Memory in retrieval of geometrical facts and geometrical processing: retrieval geometrical facts, remembering theorems, remembering hypothesis and thesis which are focusing on.

Table 1: Karagiannakis's and colleagues' frame: domains of the four-pronged model and sets of mathematical skills associated with each domain

Domain	Mathematical skills associated with the domain
Core number	Estimating accurately a small number of objects (up to 4); estimating approximately quantities; placing numbers on number lines; managing Arabic symbols; transcoding a number from one representation to another (analogical-Arabic-verbal); counting principles awareness
Memory (retrieval and processing)	Retrieving numerical facts; decoding terminology (numerator, denominator, isosceles, equilateral); remembering theorems and formulas; performing mental calculations fluently; remembering procedures and keeping track of steps
Reasoning	Grasping mathematical concepts, ideas and relations; understanding multiple steps in complex procedures/algorithms; grasping basic logical principles (conditionality – "if … then …" statements – commutativity, inversion); grasping the semantic structure of problems; (strategic) decision-making; generalizing
Visual-spatial	Interpreting and using spatial organization of representations of mathematical objects (for example, numbers in decimal positional notation, exponents, geometrical 2D and 3D figures or rotations); placing numbers on a number line; confusing Arabic numerals and mathematics symbols; performing written calculation when position is important (e.g. borrowing/carrying); interpreting graphs and tables

Since this Intervention tool concerns geometrical activity, we consider the Duval's theory on different cognitive apprehensions of figures, as the way to see, construct and describe a geometrical figure and its properties.
The Duval model is of particular interest as it is concerned with understanding the development of cognitive processes as revealed when solving geometry problems (Duval, 1998). Duval (1995) suggests an analytic theory for analysing thinking processes involved in a geometric activity.
As matter of fact, in Duval's cognitive model of geometrical reasoning, the figure plays a key role:
• A figure gives us a figural representation of a geometrical situation which is shorter and easier to be understood than a representation with linguist speech.

2 For a complete list of the principles, guidelines and checkpoints and a more extensive description of CAST's activities, visit http://www.udlcenter.org
3 The items are taken from the interactive list at http://www.udlcenter.org/research/researchevidence

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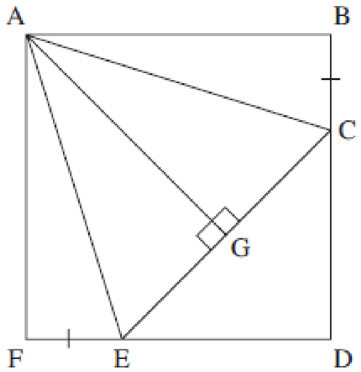
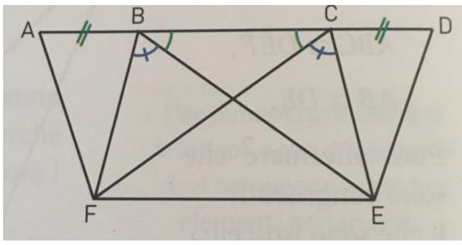
• There are different cognitive apprehensions of figures through which Seeing, constructing and describing a geometrical figure and its properties:

1. Perceptual apprehension
2. Sequential apprehension
3. Discursive apprehension
4. Operative apprehension

1. Perceptual apprehension: It is about physical recognition (shape, representation, size, brightness, etc.) of a perceived figure. We should also discriminate and recognize sub-figures within the perceived figures since a relevant discrimination or recognition of these sub-figure units may help and give cues for problem solving in geometrical situations.



Or the following figure:



For example, the sub-figure FBE and FCE that are also superposed.

2. Sequential apprehension: It is about construction of a figure or description of its construction. Such construction depends on technical constraints and also mathematical properties since construction of a figure may merge different figural units. It is believed that construction can help recognition of relationships between mathematical properties and technical constraints.
3. Discursive apprehension: It is about (a) the ability to connect configuration(s) with geometric principles, (b) the ability to provide good description, explanation, argumentation, deduction, use of symbols, reasoning depending on statements made on perceptual apprehension, and (c) the ability to describe figures through geometric language/narrative texts
4. Operative apprehension : It is about making modification of a given figure in various ways to investigate others configurations:
- The mereological way: dividing the whole given figure into parts of various shapes and combine these parts in another figure or sub-figures;
- The optic way: varying the size of the figures; you can make a shape larger or narrower, or slant, the shapes can appear differently;
- The place way: varying the position or its orientation.

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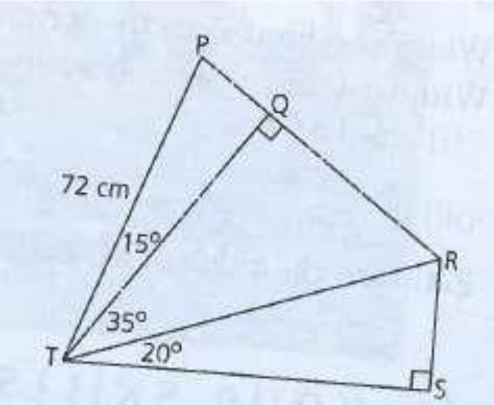


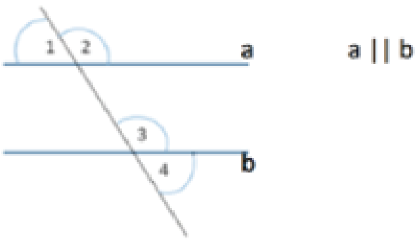
3. Design

3.1 Difficulties identified through the B2 questionnaire

We detect difficulties in the following item of B2:

2. The sum of the interior angles of a triangle is equal to…..

3.



Which sentences are true?

A. Angles 1 and 4 are equals
B. Angles 2 and 3 have the sum of 180°
C. Angles 1 and 2 have the sum of 180°
D. Angle 3 is greater than angle 2

Difficulties are related to:

- Recover from memory of theorems
- visualize and interpret information on the drawing (visualize angles and interpret their numerical code)

3.2 Cognitive area and math domain of interest

The area of difficulties identified through the B2 questionnaire is related to the domain of *Geometry*. *Memory* is the cognitive area involved.
In Table 1 the location of difficulties with respect to cognitive domain and mathematical area.

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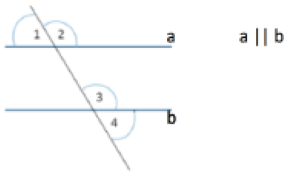
Table 1: The difficulties detected are linked to the cognitive domain of *Memory* and in the domain of *Geometry*

	Arithmetic	**Geometry**	**Algebra**
Memory		2. The sum of the interior angles of a triangle is equal to …. 3.  Which sentences are true? A. Angles 1 and 4 are equal B. Angles 2 and 3 have the sum 180° C. Angles 1 and 2 have the sum 180° D. Angle 3 is greater than angle 2	
Reasoning			
Visuo-spatial			

3.3 Educational Aims
The intervention tool is aimed at *Constructing strategies in order to retrieve geometric facts and keep them in memory in order to use them for reasoning*

3.4 Addressing to Student /class
The Intervention tool is articulated in an activity that can to be carried out with the student or all the class.

3.5 Educational activities: the Intervention Tool
The starting point concerning the design of educational activities consists in the following statement: the way a text of a task is presented (for instance, a text requiring to demonstrate a geometric theorem), conditions the working memory and the ability to recover from the memory of information (UDL principles).

In a geometrical proof task, memory is involved in order to:
- retrieve theorems and information
- keep in mind hypotheses (presented into the text of the task)
- structure a plan to demonstrate

The educational activity of this intervention tool is conceived to support metacognition. It promotes the development of strategies which allow students to support memory in its different function mentioned above. In particular, this activity focuses on that is to retrieve theorem (in order to infer information to be used).

To memorize and retrieve theorems and information
To memorize and retrieve theorems and information, it seems to be useful to construct their meaning by dynamic representations, for instance through GeoGebra functionalities. As matter of fact, a

GeoGebra construction supports abilities to recognize similarities in proposed configurations (geometrical figure) with the figure associated to the theorem.

TASK 1

With the aim to visualize theorem, by showing the relationships between angles formed by a pair of parallel lines and a transversal, this task exploit GeoGebra representations.
In Figure 2 you can find a screenshot captured from the GeoGebra site (https://www.geogebra.org/m/rSuyACJC) where drag function and dynamicity of GeoGebra drwing are exploited to visualize and act on the geometrical figure.

The aim of the task is to use the drag function to display different configurations associated with the theorem. This allows to shape the idea of generalization of the geometric figure associated with the theorem and allows you to create a sort of data base of possible images to be recalled to identify in the drawing provided, the configuration of the theorem of parallel straight lines cut by a transverse one (useful for purposes of proof).

Step 1

This applet shows the relationships between the couple of angles formed by a pair of parallel lines and a transversal. The perception of this regularity is supported both by dragging lines and change angles. In the first case, lines change their inclination but they maintain parallelism between them. In the second case, angles change their angular size but they maintain congruence between them.

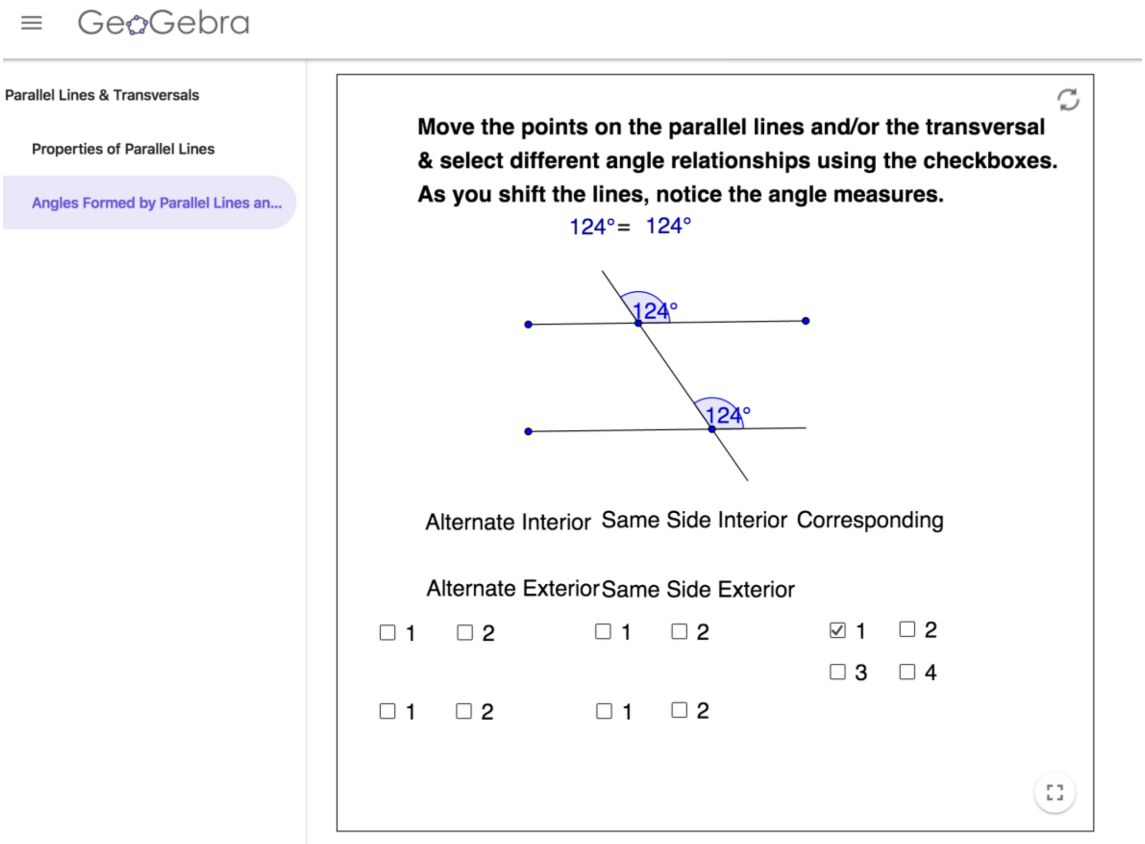


Figure 2: (dynamic) verification of the relationships between a couple of angles formed by a pair of parallel lines and a transversal

In Figure 3 you can see a screenshot captured from the GeoGebra site (https://www.geogebra.org/m/rSuyACJC#material/R6by3Bu) where are exploited drag function and dynamicity of the geometrical figure.

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Step 2

Lines AB and FC are parallel. Line BC is a transversal of the two parallel lines AB and FC. Use points A, B, and C to change the angles values. When a transversal intersects two parallel lines, what angle relationships are formed? Make as many observations as you can.

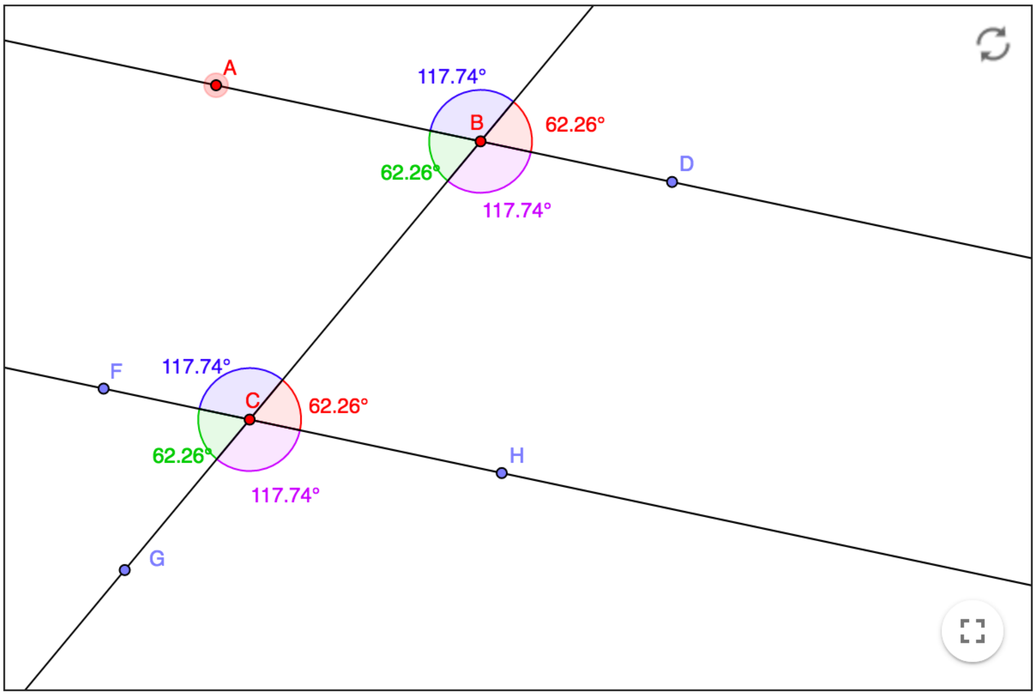

Figure 3: (dynamic) verification of the relationships between angles formed by a pair of parallel lines and a transversal

Note that colours allow students to focus their attention on a couple of angles bypassing the numerical code.

Discussion through UDL guidelines about the above-mentioned activities

In *red* our comments to illustrate the connection between the principles of UDL and our activities.

Table 3: Analysis of the activities through the Table of UDL principles.

Engagement	*Representation*	*Action & Expression*
Recruiting interest *Optimize individual choice and autonomy* *Optimize relevance, value, and authenticity* *Minimize threats and distractions*	***Perception*** Offer ways of customizing the display of information Offer alternatives for auditory information Offer alternatives for visual information Different registers through which information are displayed (visual non verbal, verbal and symbolic)	***Physical Action*** Vary the methods for response and navigation Optimize access to tools and assistive technologies *Geogebra allows students physical action on the figural objects and gives them appropriate feedbacks on their action*
Sustaining effort Persistence	***Language & Symbols*** Clarify vocabulary and	***Expression Communication***

Heighten salience of goals and objectives Vary demands and resources to optimize challenge Foster collaboration and community Increase mastery-oriented feedback Vary demands and resources to optimize challenge Foster collaboration and community *Feedbacks of software support engagement and motivation with respect the elaboration of the solution of the task*	symbols Clarify syntax and structure Offer alternative language and symbols to decode information and to work on the information *This is promoted by the use of different registers of representation: figural non verbal on the drawing, colors* Support decoding of text, mathematical notation, and symbols Promote understanding across languages Illustrate through multiple media *This is promoted by the use of dynamic geometry software such as GeoGebra.* Support decoding of text, math notation and symbols *This is promoted by the visualization of hypotheses by drawing realized by GeoGebra*	Use multiple media for communication Use multiple tools for construction and composition Build fluencies with graduated levels of support for practice and performance To use different registers in order to communicate
Self Regulation *Promote expectations and beliefs that optimize motivation* *Facilitate personal coping skills and strategies* Develop self-assessment and reflection	***Comprehension*** Activate or supply background knowledge Highlight patterns, critical features, big ideas, and relationships (checkpoint 3.2) Guide information processing and visualization Maximize transfer and generalization *To support generalization, the tasks suggest to visualize drawings on GeoGebra. Indeed, the drag function of GeoGebra allows students to identify invariants of the*	***Executive functions*** *Guide appropriate goal-setting* *The use drag funztion to visualize invariant elements on the figure allows student to construct a sort of data base of images. They support memory to identify, in the provided drawing, the configuration of the theorem that have to be used.* *Support planning and strategy development* *Facilitate managing information and resources* *Enhance capacity for monitoring progress*

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	figure and recover suitable theorem in order to develop the required proof. Perception, language and symbols, comprehension (Constructing useable knowledge, knowledge that is accessible for future decision-making, depends not upon merely perceiving information, but upon active “information processing skills”)	

4. References

[1] Karagiannakis, G. N., Baccaglini-Frank, A. E., & Roussos, P. (2016). Detecting strengths and weaknesses in learning mathematics through a model classifying mathematical skills. Australian J. of Learning Difficulties, 21(2), 115–141.

[2] Duval, R.: 1995, ‘Geometrical Pictures: Kinds of representation and specific processing’, in R. Suttherland and J. Mason (eds.), Exploiting Mental Imagery with Computers in Mathematics Education, Springer, Berlin, pp. 142–157.

[3] Duval, R.: 1998, ‘Geometry from a cognitive point a view’, in C. Mammana and V. Villani (eds.), Perspectives on the Teaching of Geometry for the 21st Century, Kluwer Academic Publishers, Dordrecht, pp. 37–52.

[4] UDL Principles: http://udlguidelines.cast.org/

[5] GeoGebra materials:https://www.geogebra.org/m/rSuyACJC

[6] https://www.geogebra.org/m/rSuyACJC#material/R6by3BuA