Project Number: 2018-1-IT02-KA201-048274

INTERVENTION TOOL

Supporting Memory in Geometrical Demonstration Process

University of Genova[1]

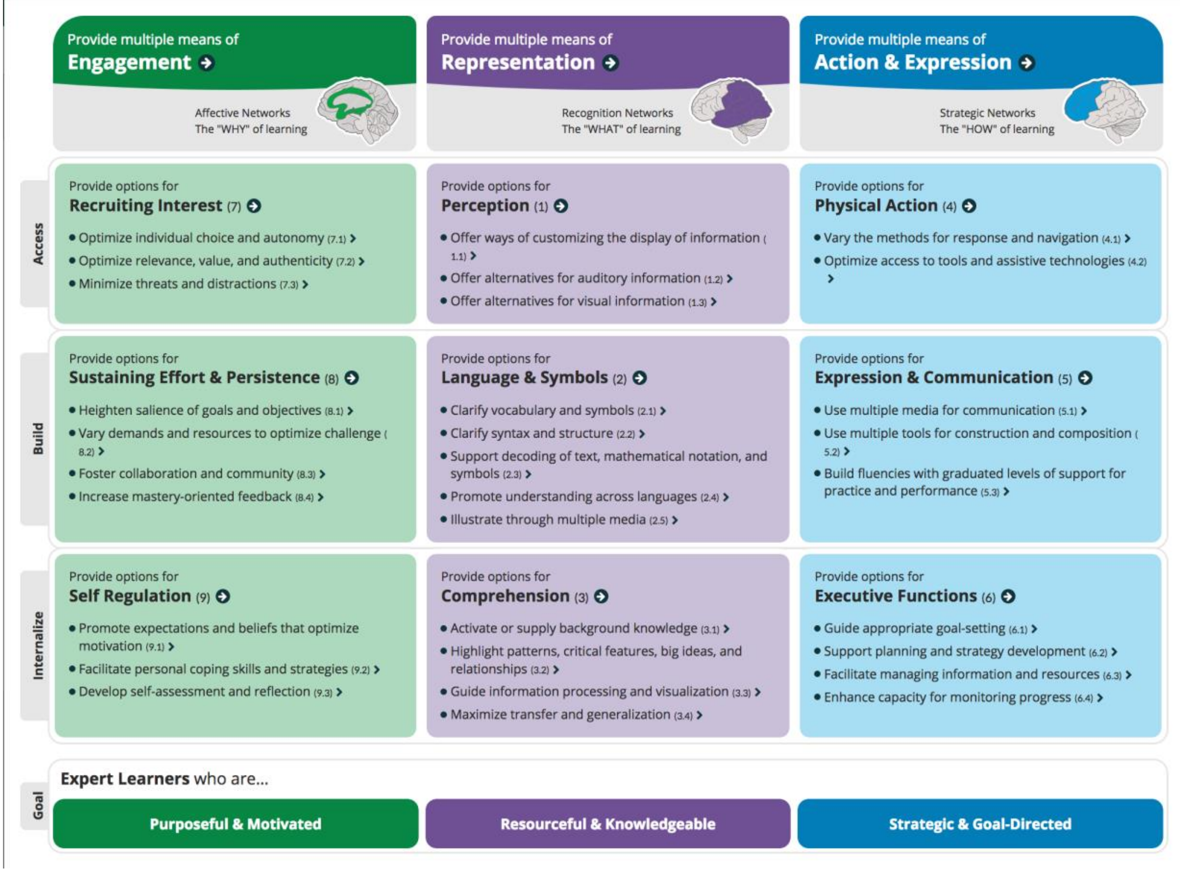
1. Introduction

In order to develop educational activities aimed to support memory in geometry, we refer to some theoretical frameworks that will be described in the session 2.
In session 3 the design of the educational activities is described. In particular, if the activities are addressed to students or the class, the educational aim of the activities, the Cognitive area and math domain of interest and the Mathematical objects in the areas of difficulties identified through the B2 questionnaire

2. Theoretical framework of reference

The theoretical references that helped us to design the following activities are:
1) **Universal design for learning (UDL) principles** (Table 3), a framework specifically conceived to design *inclusive* educational activities (http://udlguidelines.cast.org/)

Table 3: UDL guidelines



	Provide multiple means of **Engagement**	Provide multiple means of **Representation**	Provide multiple means of **Action & Expression**
	Affective Networks The "WHY" of learning	Recognition Networks The "WHAT" of learning	Strategic Networks The "HOW" of learning
Access	Provide options for **Recruiting Interest** (7) • Optimize individual choice and autonomy (7.1) • Optimize relevance, value, and authenticity (7.2) • Minimize threats and distractions (7.3)	Provide options for **Perception** (1) • Offer ways of customizing the display of information (1.1) • Offer alternatives for auditory information (1.2) • Offer alternatives for visual information (1.3)	Provide options for **Physical Action** (4) • Vary the methods for response and navigation (4.1) • Optimize access to tools and assistive technologies (4.2)
Build	Provide options for **Sustaining Effort & Persistence** (8) • Heighten salience of goals and objectives (8.1) • Vary demands and resources to optimize challenge (8.2) • Foster collaboration and community (8.3) • Increase mastery-oriented feedback (8.4)	Provide options for **Language & Symbols** (2) • Clarify vocabulary and symbols (2.1) • Clarify syntax and structure (2.2) • Support decoding of text, mathematical notation, and symbols (2.3) • Promote understanding across languages (2.4) • Illustrate through multiple media (2.5)	Provide options for **Expression & Communication** (5) • Use multiple media for communication (5.1) • Use multiple tools for construction and composition (5.2) • Build fluencies with graduated levels of support for practice and performance (5.3)
Internalize	Provide options for **Self Regulation** (9) • Promote expectations and beliefs that optimize motivation (9.1) • Facilitate personal coping skills and strategies (9.2) • Develop self-assessment and reflection (9.3)	Provide options for **Comprehension** (3) • Activate or supply background knowledge (3.1) • Highlight patterns, critical features, big ideas, and relationships (3.2) • Guide information processing and visualization (3.3) • Maximize transfer and generalization (3.4)	Provide options for **Executive Functions** (6) • Guide appropriate goal-setting (6.1) • Support planning and strategy development (6.2) • Facilitate managing information and resources (6.3) • Enhance capacity for monitoring progress (6.4)
Goal	**Expert Learners** who are... **Purposeful & Motivated**	**Resourceful & Knowledgeable**	**Strategic & Goal-Directed**

The Center for Applied Special Technology (CAST) has developed a comprehensive framework around the concept of Universal Design for Learning (UDL), with the aim of focusing research,

[1] Emanuela De Negri, Elisabetta Robotti, Francesca Morselli, Paola Viterbori, Anna Siri, Laura Capelli

development, and educational practice on understanding diversity and facilitating learning (Edyburn, 2005). UDL includes a set of Principles, articulated in *Guidelines and Checkpoints*[2]. The research grounding UDL's framework is that "learners are highly variable in their response to instruction. [...]"
Thus, UDL focus on these individual differences as an important element to understanding and designing effective instruction for learning.
To this aim, UDL advances three foundational Principles: 1) provide multiple means of representation, 2) provide multiple means of action and expression, 3) provide multiple means of engagement. In particular, guidelines within the first principle have to do with means of perception involved in receiving certain information, and of "comprehension" of the information received. Instead, the guidelines within the second principle take into account the elaboration of information/ideas and their expression. Finally, the guidelines within the third principle deal with the domain of "affect" and "motivation", also essential in any educational activity.
For our analyses we will focus in particular on specific guidelines within the three Principles[3].
In order to characterize students' difficulties in geometry, we refer to the following elements of Karagiannakis' and colleagues' frame (Table 1), which dealt with Memory in retrieval of geometrical facts and geometrical processing: retrieval geometrical facts, remembering theorems, remembering hypothesis and thesis which are focusing on.

Table 1: Karagiannakis's and colleagues' frame: domains of the four-pronged model and sets of mathematical skills associated with each domain

Domain	Mathematical skills associated with the domain
Core number	Estimating accurately a small number of objects (up to 4); estimating approximately quantities; placing numbers on number lines; managing Arabic symbols; transcoding a number from one representation to another (analogical-Arabic-verbal); counting principles awareness
Memory (retrieval and processing)	Retrieving numerical facts; decoding terminology (numerator, denominator, isosceles, equilateral); remembering theorems and formulas; performing mental calculations fluently; remembering procedures and keeping track of steps
Reasoning	Grasping mathematical concepts, ideas and relations; understanding multiple steps in complex procedures/algorithms; grasping basic logical principles (conditionality – "if … then …" statements – commutativity, inversion); grasping the semantic structure of problems; (strategic) decision-making; generalizing
Visual-spatial	Interpreting and using spatial organization of representations of mathematical objects (for example, numbers in decimal positional notation, exponents, geometrical 2D and 3D figures or rotations); placing numbers on a number line; confusing Arabic numerals and mathematics symbols; performing written calculation when position is important (e.g. borrowing/carrying); interpreting graphs and tables

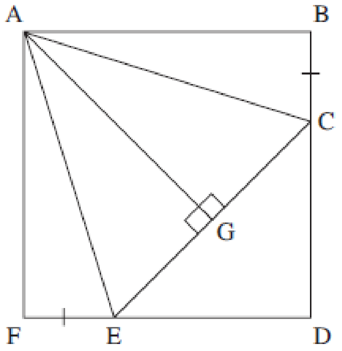
Since this Intervention tool concerns geometrical activity, we consider the Duval's theory on different cognitive apprehensions of figures, as the way to see, construct and describe a geometrical figure and its properties.
The Duval model is of particular interest as it is concerned with understanding the development of cognitive processes as revealed when solving geometry problems (Duval, 1998). Duval (1995) suggests an analytic theory for analysing thinking processes involved in a geometric activity.
As matter of fact, in Duval's cognitive model of geometrical reasoning, the figure plays a key role:

- A figure gives us a figural representation of a geometrical situation which is shorter and easier to be understood than a representation with linguist speech.
- There are different cognitive apprehensions of figures through which Seeing, constructing and describing a geometrical figure and its properties:
    1. Perceptual apprehension
    2. Sequential apprehension
    3. Discursive apprehension

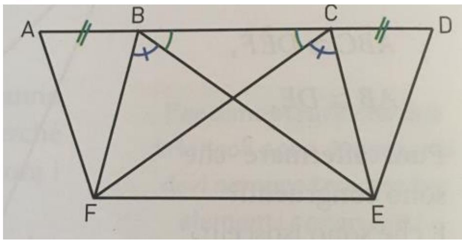
[2] For a complete list of the principles, guidelines and checkpoints and a more extensive description of CAST's activities, visit http://www.udlcenter.org
[3] The items are taken from the interactive list at http://www.udlcenter.org/research/researchevidence

4. Operative apprehension

1. Perceptual apprehension: It is about physical recognition (shape, representation, size, brightness, etc.) of a perceived figure. We should also discriminate and recognize sub-figures within the perceived figures since a relevant discrimination or recognition of these sub-figure units may help and give cues for problem solving in geometrical situations.



Or the following figure:



For example, the sub-figure FBE and FCE that are also superposed.

2. Sequential apprehension: It is about construction of a figure or description of its construction. Such construction depends on technical constraints and also mathematical properties since construction of a figure may merge different figural units. It is believed that construction can help recognition of relationships between mathematical properties and technical constraints.
3. Discursive apprehension: It is about (a) the ability to connect configuration(s) with geometric principles, (b) the ability to provide good description, explanation, argumentation, deduction, use of symbols, reasoning depending on statements made on perceptual apprehension, and (c) the ability to describe figures through geometric language/narrative texts
4. Operative apprehension : It is about making modification of a given figure in various ways to investigate others configurations:

- The mereological way: dividing the whole given figure into parts of various shapes and combine these parts in another figure or sub-figures;
- The optic way: varying the size of the figures; you can make a shape larger or narrower, or slant, the shapes can appear differently;
- The place way: varying the position or its orientation.

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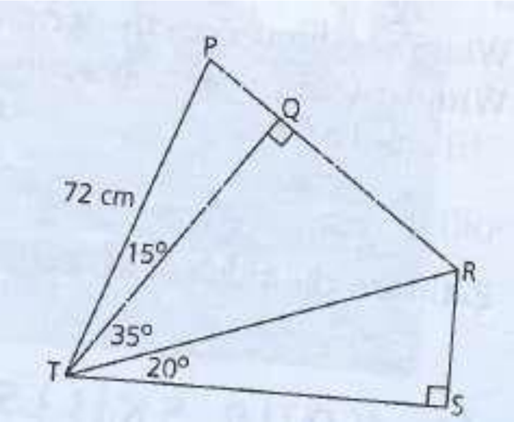
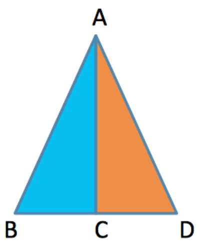


3. Design

3.1 Difficulties identified through the B2 questionnaire

We detect difficulties in the following items of B2 related to Geometry domain:

1)



BC=CD, BA=AD

Which kind of triangle is CDA?

Which kind of triangle is BDA?

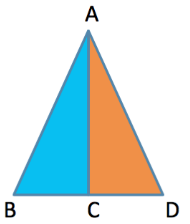
Difficulties are related to :

- shift constantly visual focus from the written text to the drawing and vice versa in order to keep in mind information obtained form the text
- Visualize on the drawing the geometrical information detected from hypothesis
- Process those information (use them in proof)

3.2 Cognitive area and math domain of interest

The area of difficulties identified through the B2 questionnaire is related to the domain of *Geometry*. *Memory* is the cognitive area involved.
In Table 2 the **location of difficulties** with respect to cognitive domain and mathematical area.

Table 2: The difficulties detected are linked to the cognitive domain of *Memory* and in the domain of *Geometry*

	Arithmetic	Geometry	Algebra
Memory		 BC=CD, BA=AD Which kind of triangle is CDA? Which kind of triangle is BDA?	
Reasoning			
Visuo-spatial			

3.3 Educational Aims

The intervention tool is aimed at *Constructing strategies in order to retrieve geometric facts and theorems or properties, keep them in memory and use them for reasoning.*

3.4 Addressing to Student /class

The Intervention tool is articulated in an activity that have to be carried out with the student or all the class.

3.5 Educational activities: the Intervention Tool

The starting point concerning the design of this kind of educational activity consists in the following statement: the way a text of a task is presented (for instance, a text requiring to proof a geometrical theorem), affects the working memory and the ability to recover from the memory of information (UDL principles).

In a geometrical proof task, memory is involved in order to:

- retrieve theorems and information (see Interventional tool *“Supporting memory in remember theorem”)*
- keep in mind hypotheses (presented into the text of the task)
- structure a plan to proof

The educational activities of this intervention tool are conceived to support metacognition. They promote the development of strategies that allow students to support memory in its different function: to retrieve theorems (in order to infer information to be used), to keep in mind information obtained form the text (hypothesis), and to process information in order to reach the required thesis (in this regards, see also Intervention tools located in Reasoning area and Geometry domain).

To keep in mind hypotheses (presented into the text of the task)

According to Duval (1995), a drawing, on some form of support, in geometrical context gives us a figural representation of a geometrical situation, which is shorter and easier to be understood than a representation with linguist speech.

Thus, to avoid constantly shifting student’s visual focus from the written text to the drawing in order to keep in mind information obtained form the text, teacher promote the visualization on the drawing of the information obtained in verbal form through the text. The student is asked to *rewrite the* *hypotheses directly on the drawing by choosing appropriate graphic signs* in visual non-verbal code. The signs refer to meanings related to hypotheses. Thus, the transcoding of information is required.

Teacher introduces different graphical signs to rewriting information about geometrical figure (or, more in general, about hypothesis) in visual non-verbal form.

In other words, the teacher promotes transcoding process.
As matter of fact, once student identified hypotheses (to this regard, see also Intervention tools on Reasoning in Geometry) on the task text, he/she have to translate them on the drawing in order to process them by retrieval geometrical fact (theorem). These meta-activities involve necessarily visual-spatial aspects. For this, we suggest to consider also the Intervention tools referring to the Visuo-spatial area in Geometry domain.

The main semiotic registers involved concern:

- Graphical signs on the drawing
- Colours
- Use of concrete material to support visualization (such as colored transparent sheets Figure 1)

Figure 1: transparent coloured figures

Example of Tasks

TASK 1

Consider the isosceles triangle ABD, and C is a point on BD that BC=CD. Which kind of triangle is CDA? Which kind of triangle is BDA?

Note that text of the task is exclusively in verbal register.

Step 1: identifying hypothesis

- Isosceles triangle ABD
- C is a point on BD that BC=CD
- A is vertex of triangle opposite to the base BD (implicit hypothesis)

Step 2: Processing information and retrieving theorem

- Drawing an isosceles triangle with base BD
- Isosceles triangle ABD → AD=AB and angle in B is equal to the angle in D [this is done by remembering theorem about Isosceles triangle].
- C is a point on BD that BC=CD → C is the midpoint of BD → C is the midpoint of BD → C is the foot of its altitude related to the base BD [this is done by remembering theorem about altitude in Isosceles triangle] →AC altitude of the triangle, related to BD.

Step 3: Transcoding process

1) Isosceles triangle ABD → AD=AB and angle in B is equal to the angle in D

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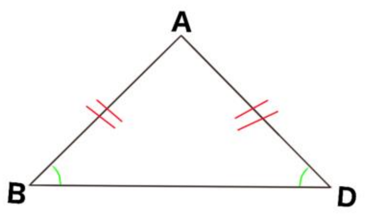
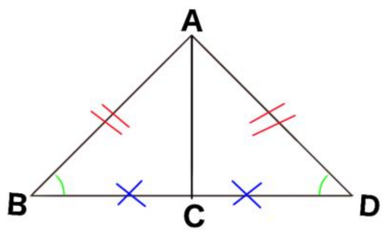
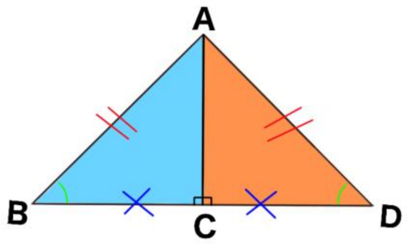


- C is a point on BD that BC=CD → C is the midpoint of BD → C is the middle point of BD → C is the foot of its altitude related to the base BD →AC altitude related to BD of the triangle



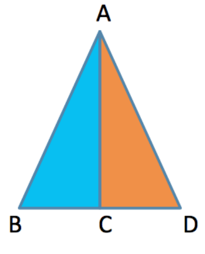
Step 4
Process drawing in order to proof thesis:
Which kind of triangle is CDA? Which kind of triangle is BDA?



Note that colours allow students to identify perceptively the sub-figures on which reasoning has to be focused. In other words, colours allow students to perform operative apprehension (Duval , 1995) to investigate others Configurations.

TASK 2

Consider the triangle ABD in the drawing,



BC=CD, BA=AD

Which kind of triangle is CDA? Which kind of triangle is BDA?

Project Number: 2018-1-IT02-KA201-048274

Step 1: identify hypothesis

The hypotheses have to be identified in the text (triangle ABD), in the drawing added to the text (blue and orange triangles and the global triangle obtained through orange triangle near the blue one) and in verbal code added to the drawing (BC=CD, BA=AD). Thus, verbal code and figural code present the following hypotheses:

There is a “big” triangle (named ABD), this is an isosceles triangle, in this triangle is present the median related to the basis, it defines two triangles (blue and orange)

We can observe that, in the case of black and white drawing, the visualization of the two triangles (BCA and DCA) is not perceptively evident as in the case of coloured triangles.

Thus the use of colours became an important tool in order to:

- Present information (hypothesis);
- Process information.

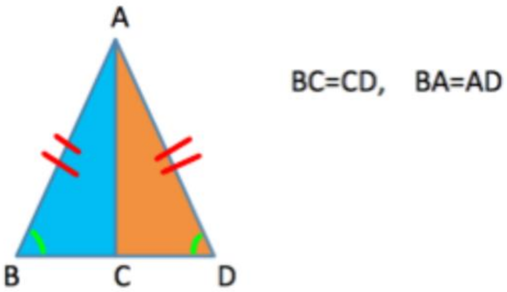
This concerns the visuo-spatial domain because students interpret and use the spatial organization of coloured representations of triangles in order to processing information and develop reasoning on the geometrical task (see Intervention tools in Visuo-Spatial area of Geometry)

Step 2: Processing information

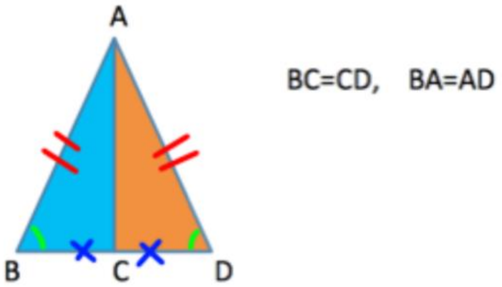
- AC is the median related to the basis BD of the Isosceles triangle ABD → AC is the altitude related to the base BD
- The side AC of the blue and orange triangles identifies two right triangles

Step 3: Transcoding process

2) BC=CD, BA=AD → BDA is isosceles triangle



3) AC is the median and altitude



4) Blue and orange triangles are right triangles

BC=CD, BA=AD

Discussion through UDL guidelines about the above-mentioned activities

In *red* our comments to illustrate the connection between the principles of UDL and the described tasks.

Table 3: Analysis of the activities through the Table of UDL principles.

Engagement	*Representation*	*Action & Expression*
Recruiting interest *Optimize individual choice and autonomy* *Optimize relevance, value, and authenticity* *Minimize threats and distractions*	***Perception*** Offer ways of customizing the display of information Offer alternatives for auditory information Offer alternatives for visual information Different registers through which information are displayed (visual non verbal, verbal and symbolic)	***Physical Action*** Vary the methods for response and navigation Optimize access to tools and assistive technologies *Geogebra allows students physical action on the figural objects and gives them appropriate feedbacks on their action*
Sustaining effort Persistence Heighten salience of goals and objectives Vary demands and resources to optimize challenge Foster collaboration and community Increase mastery-oriented feedback Vary demands and resources to optimize challenge Foster collaboration and	***Language & Symbols*** Clarify vocabulary and symbols Clarify syntax and structure Offer alternative language and symbols to decode information and to work on the information *This is promoted by the use of different registers of representation: figural non verbal on the drawing* Support decoding of text, mathematical notation, and symbols	***Expression Communication*** Use multiple media for communication Use multiple tools for construction and composition Build fluencies with graduated levels of support for practice and performance To use different registers in order to communicate *Figural register both in static (Sheet of paper, transparent*

community *Oriented feedbacks support engagement and motivation with respect the elaboration of the solution of the task*	Promote understanding across languages Illustrate through multiple media *This is promoted by the use of dynamic geometry software such as GeoGebra.* Support decoding of text, math notation and symbols *This is promoted by the visualization of hypotheses by the figural non-verbal register and by the dynamic figure on GeoGebra*	*coloured sheets, …) or dynamic environment (GeoGebra) provides students of different register to communicate and also to process informations.* *This allows them to manage in more efficient way executive functions.*
Self Regulation *Promote expectations and beliefs that optimize motivation* *Facilitate personal coping skills and strategies* Develop self-assessment and reflection	***Comprehension*** Activate or supply background knowledge Highlight patterns, critical features, big ideas, and relationships (checkpoint 3.2) Guide information processing and visualization *To process information and hypotheses, tasks suggest to visualize them directly oon the drawing* Maximize transfer and generalization *To support generalization, the tasks suggest to visualize drawings on GeoGebra. Indeed, the drag function of GeoGebra allows students to identify invariants of the figure and recover suitable theorem in order to develop the required proof.* Perception, language and symbols, comprehension (Constructing useable knowledge, knowledge that is accessible for future decision-making, depends not upon merely perceiving information, but upon active “information processing skills”)	***Executive functions*** *Guide appropriate goal-setting* *The use of visual register of representation to visualize hypotheses on the drawing may be a support for memory. This could support students’ process of retrieval theorems and processes* *Support planning and strategy development* *Facilitate managing information and resources* *Enhance capacity for monitoring progress*

4. References

[1] Duval, R.: (1995), Geometrical Pictures: Kinds of representation and specific processing, in R. Suttherland and J. Mason (eds.), Exploiting Mental Imagery with Computers in Mathematics Education, Springer, Berlin, pp. 142–157.

[2] Duval, R. (1998), Geometry from a cognitive point a view, in C. Mammana and V. Villani (eds.), Perspectives on the Teaching of Geometry for the 21st Century, Kluwer Academic Publishers, Dordrecht, pp. 37–52.

[3] Karagiannakis, G. N., Baccaglini-Frank, A. E., & Roussos, P. (2016). Detecting strengths and weaknesses in learning mathematics through a model classifying mathematical skills. Australian J. of Learning Difficulties, 21(2), 115–141.

[4] UDL Principles: http://udlguidelines.cast.org/

[5] GeoGebra materials: https://www.geogebra.org/m/rSuyACJC

[6] https://www.geogebra.org/m/rSuyACJC#material/R6by3BuA

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