SMILD

Project Number: 2018-1-IT02-KA201-048274

INTERVENTION TOOL

Supporting Visual-Spatial in Geometry

University of Genova[1]

1. Introduction

In order to develop educational activities aimed to support visuo-spatial in geometry, we refer to some theoretical frameworks that will be described in the session 2.
In session 3 the design of the educational activities is described. In particular, if the activities are addressed to students or the class, the educational aim of the activities, the Cognitive area and math domain of interest and the Mathematical objects in the areas of difficulties identified through the B2 questionnaire

2. Theoretical framework of reference

The theoretical references that helped us to design the following activities are:

1) **Universal design for learning (UDL) principles** (Table 3), a framework specifically conceived to design *inclusive* educational activities (http://udlguidelines.cast.org/)

Table 3: UDL guidelines

	Provide multiple means of **Engagement** → Affective Networks The "WHY" of learning	Provide multiple means of **Representation** → Recognition Networks The "WHAT" of learning	Provide multiple means of **Action & Expression** → Strategic Networks The "HOW" of learning
Access	Provide options for **Recruiting Interest** (7) • Optimize individual choice and autonomy (7.1) › • Optimize relevance, value, and authenticity (7.2) › • Minimize threats and distractions (7.3) ›	Provide options for **Perception** (1) • Offer ways of customizing the display of information (1.1) › • Offer alternatives for auditory information (1.2) › • Offer alternatives for visual information (1.3) ›	Provide options for **Physical Action** (4) • Vary the methods for response and navigation (4.1) › • Optimize access to tools and assistive technologies (4.2) ›
Build	Provide options for **Sustaining Effort & Persistence** (8) • Heighten salience of goals and objectives (8.1) › • Vary demands and resources to optimize challenge (8.2) › • Foster collaboration and community (8.3) › • Increase mastery-oriented feedback (8.4) ›	Provide options for **Language & Symbols** (2) • Clarify vocabulary and symbols (2.1) › • Clarify syntax and structure (2.2) › • Support decoding of text, mathematical notation, and symbols (2.3) › • Promote understanding across languages (2.4) › • Illustrate through multiple media (2.5) ›	Provide options for **Expression & Communication** (5) • Use multiple media for communication (5.1) › • Use multiple tools for construction and composition (5.2) › • Build fluencies with graduated levels of support for practice and performance (5.3) ›
Internalize	Provide options for **Self Regulation** (9) • Promote expectations and beliefs that optimize motivation (9.1) › • Facilitate personal coping skills and strategies (9.2) › • Develop self-assessment and reflection (9.3) ›	Provide options for **Comprehension** (3) • Activate or supply background knowledge (3.1) › • Highlight patterns, critical features, big ideas, and relationships (3.2) › • Guide information processing and visualization (3.3) › • Maximize transfer and generalization (3.4) ›	Provide options for **Executive Functions** (6) • Guide appropriate goal-setting (6.1) › • Support planning and strategy development (6.2) › • Facilitate managing information and resources (6.3) › • Enhance capacity for monitoring progress (6.4) ›
Goal	**Expert Learners** who are... **Purposeful & Motivated**	**Resourceful & Knowledgeable**	**Strategic & Goal-Directed**

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The Center for Applied Special Technology (CAST) has developed a comprehensive framework around the concept of Universal Design for Learning (UDL), with the aim of focusing research, development, and educational practice on understanding diversity and facilitating learning (Edyburn, 2005). UDL includes a set of Principles, articulated in *Guidelines and Checkpoints*[2]. The research grounding UDL's framework is that "learners are highly variable in their response to instruction. [...]"
Thus, UDL focus on these individual differences as an important element to understanding and designing effective instruction for learning.
To this aim, UDL advances three foundational Principles: 1) provide multiple means of representation, 2) provide multiple means of action and expression, 3) provide multiple means of engagement. In particular, guidelines within the first principle have to do with means of perception involved in receiving certain information, and of "comprehension" of the information received. Instead, the guidelines within the second principle take into account the elaboration of information/ideas and their expression. Finally, the guidelines within the third principle deal with the domain of "affect" and "motivation", also essential in any educational activity.
For our analyses we will focus in particular on specific guidelines within the three Principles[3].
In order to characterize students' difficulties in geometry, we refer to the following elements of Karagiannakis' and colleagues' frame (Table 1), which dealt with Memory in retrieval of geometrical facts and geometrical processing: retrieval geometrical facts, remembering theorems, remembering hypothesis and thesis which are focusing on.

Table 1: Karagiannakis's and colleagues' frame: domains of the four-pronged model and sets of mathematical skills associated with each domain

Domain	Mathematical skills associated with the domain
Core number	Estimating accurately a small number of objects (up to 4); estimating approximately quantities; placing numbers on number lines; managing Arabic symbols; transcoding a number from one representation to another (analogical-Arabic-verbal); counting principles awareness
Memory (retrieval and processing)	Retrieving numerical facts; decoding terminology (numerator, denominator, isosceles, equilateral); remembering theorems and formulas; performing mental calculations fluently; remembering procedures and keeping track of steps
Reasoning	Grasping mathematical concepts, ideas and relations; understanding multiple steps in complex procedures/algorithms; grasping basic logical principles (conditionality – "if … then …" statements – commutativity, inversion); grasping the semantic structure of problems; (strategic) decision-making; generalizing
Visual-spatial	Interpreting and using spatial organization of representations of mathematical objects (for example, numbers in decimal positional notation, exponents, geometrical 2D and 3D figures or rotations); placing numbers on a number line; confusing Arabic numerals and mathematics symbols; performing written calculation when position is important (e.g. borrowing/carrying); interpreting graphs and tables

3. Design

3.1 Difficulties identified through the B2 questionnaire

We detect difficulties in the following item of B2:

[2] For a complete list of the principles, guidelines and checkpoints and a more extensive description of CAST's activities, visit http://www.udlcenter.org
[3] The items are taken from the interactive list at: http://www.udlcenter.org

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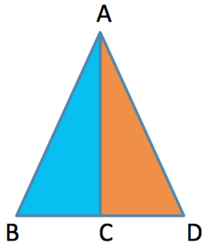


BC=CD, BA=AD

Which kind of triangle is CDA?

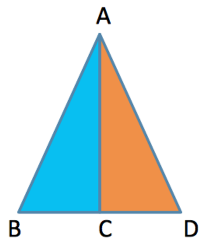
Which kind of triangle is BDA?

Difficulties are related to visualization process, which refers to the use of the figure (drawing) to illustrate the geometrical objects and to manipulate this figure in order to connect configuration(s) with geometric principles.

3.2 Cognitive area and math domain of interest

The area of difficulties identified through the B2 questionnaire is related to the domain of *Geometry*. *Visuo-spatial* is the cognitive area involved.
In Table 2 the location of difficulties with respect to cognitive domain and mathematical area.

Table 2: The difficulties detected are linked to the cognitive domain of *Visuo-spatial* and in the domain of *Geometry*

	Arithmetic	Geometry	Algebra
Memory			
Reasoning			
Visuo-spatial		 BC=CD, BA=AD Which kind of triangle is CDA? Which kind of triangle is BDA?	

3.3 Educational Aims

The intervention tool is aimed to *support visualization process that refers to the use of figures and images for illustration, exploration or verification of different geometric situations.*

3.4 Addressing to Student /class

The Intervention tool is articulated in a two activities that have to be carried out with the student or all the class.

3.5 Educational activities: the Intervention Tool

According to Duval's cognitive model of geometrical reasoning, the figure plays a key role (Duval, 1995). As matter of fact, a figure gives us a figural representation of a geometrical situation which is shorter and easier to be understood than a representation with linguist speech (to that extend, see the Intervention tool named: "*Supporting memory in geometrical demonstration process"*).

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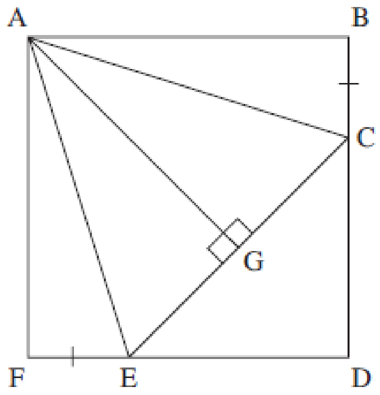
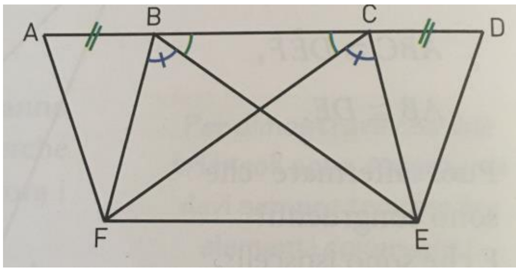
Moreover, Duval refers to different cognitive apprehensions of figures, considered as the way to see, construct and describe a geometrical figure and its properties.
1. Perceptual apprehension: It is about physical recognition (shape, representation, size, brightness, etc.) of a perceived figure. We should also discriminate and recognize sub-figures within the perceived figures since a relevant discrimination or recognition of these sub-figure units may help and give cues for problem solving in geometrical situations.



Or the following Figure:



For example, the sub-figure FBE and FCE that are also superposed.

2. Sequential apprehension: It is about construction of a figure or description of its construction. Such construction depends on technical constraints and also mathematical properties since construction of a figure may merge different figural units. It is believed that construction can help recognition of relationships between mathematical properties and technical constraints.
3. Discursive apprehension: It is about (a) the ability to connect configuration(s) with geometric principles, (b) the ability to provide good description, explanation, argumentation, deduction, use of symbols, reasoning depending on statements made on perceptual apprehension, and (c) the ability to describe figures through geometric language/narrative

texts.
4. Operative apprehension : It is about making modification of a given figure in various ways to investigate others configurations:
- The mereological way: dividing the whole given figure into parts of various shapes and combine these parts in another figure or sub-figures;
- The optic way: varying the size of the figures; you can make a shape larger or narrower, or slant, the shapes can appear differently;
- The place way: varying the position or its orientation.

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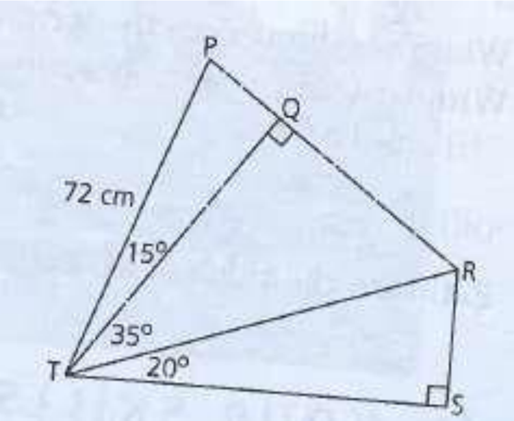

In particular, **Perceptual apprehension** and **Operative apprehension** are strictly related to visualization of figure:
For this, the educational activities of this intervention tool are conceived as a meta-activities to develop strategies which allow students to support both Perceptual apprehension and Operative apprehension

Visualizing relations through dynamic representations

The geometrical construction of a figure by digital tools such as GeoGebra seems to be effective in order to discriminate and recognize sub-figures within the perceived figures. As matter of fact, dragging function, available in these kinds of tools, allows students to identify geometrical invariants and to drag superposed figures by recognizing relationships between mathematical properties and technical constraints.
Moreover, the dynamic modification of geometrical elements allows students to identify relations among them.
For example, consider the following task:

TASK 1

In an isosceles triangle ABC determines the midpoint M of the base AB. Take a point P on the AC side and a point Q on the BC side such that it is CP≅CQ. Show that the MPQ triangle also appears to be isosceles

Students are aked to:

- Construct on GeoGebra the figure required so that it is resistent to the dragging (see construction by GeoGebra https://www.geogebra.org/m/MaUCMhcM);
- Move the points P and Q, A and B in order to identify invariants. What is asked here is the Operative apprehension of the figure by varying the size of the figures.

Note that the figure can be coloured as in Figure 1. Colour support perceptual apprehension by promoting the visualization of sub-figures and their relations

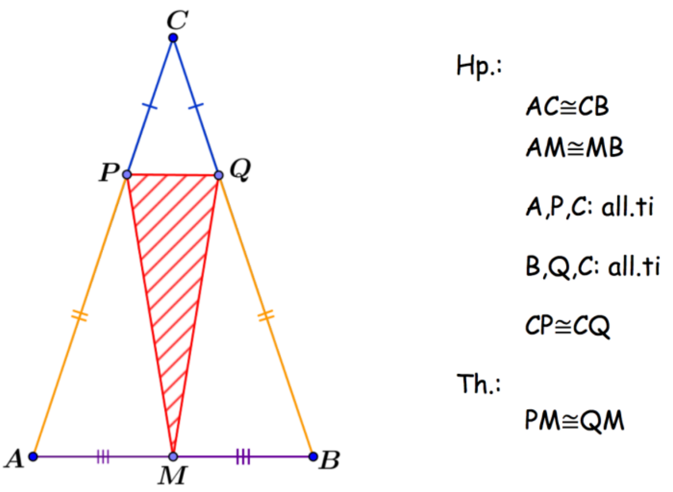

Figure 1: coloured sub-figures to support the connection of the configuration(s) with geometrical principles.

TASK 2:

Provide students with GeoGebra congruent triangles ABC and A'B'C' (Figure 2) constructed through macro so that, by changing value of AB, AC or α, students can identify invariants and visualize properties.

Students are asked to:

- Change value of AB, AC or α . This allows students to perform both Perceptual apprehension and Operative apprehension
- Describe what change and what doesn't in ABC and A'B'C'. In other words, is asked to identify invariants in the triangles ABC and A'B'C'

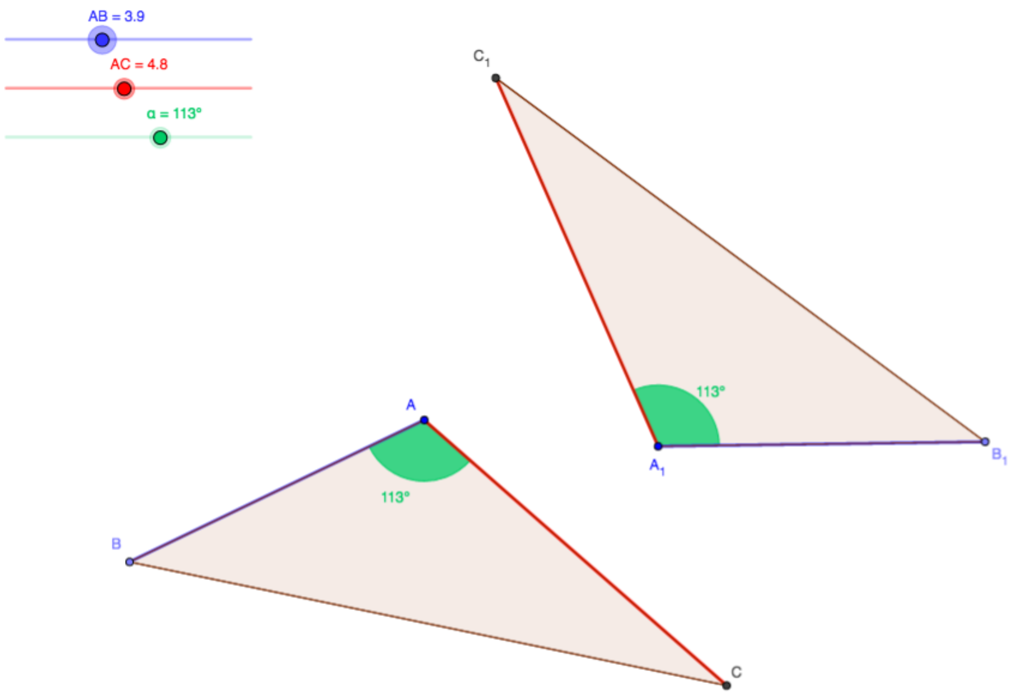

Figure 2: macro related to the first of the criteria for congruent triangles

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Note that the corresponding two congruent sides of the triangles ABC and A'B'C' are coloured in blue and red. Equally, the angles in Â and Â' are coloured in green to better identify them inside the figure.

In order to explore with GeoGebra the conditions for triangles to be congruent, see the activity at the following link:
https://www.geogebra. /m/thA4JzgH

Discussion through UDL guidelines about the above-mentioned activities

In *red* our comments to illustrate the connection between the principles of UDL and our activities.

Table 3: Analysis of the activities through the Table of UDL principles.

Engagement	***Representation***	***Action & Expression***
Recruiting interest Optimize individual choice and autonomy Optimize relevance, value, and authenticity Minimize threats and distractions	***Perception*** Offer ways of customizing the display of information Offer alternatives for auditory information Offer alternatives for visual information Different registers through which information are displayed (visual non verbal, verbal and symbolic)	***Physical Action*** Vary the methods for response and navigation Optimize access to tools and assistive technologies
Sustaining effort Persistence Heighten salience of goals and objectives Vary demands and resources to optimize challenge Foster collaboration and community Increase mastery-oriented feedback Vary demands and resources to optimize challenge Foster collaboration and community Oriented feedbacks support engagement and motivation with respect the elaboration of the solution of the task *Learning but also recognizing or the application in drawing*	***Language & Symbols*** Clarify vocabulary and symbols Clarify syntax and structure Offer alternative language and symbols to decode information and to work on the information *This is promoted by the use of differents registers of representation* Support decoding of text, mathematical notation, and symbols Promote understanding across languages Illustrate through multiple media *This is promoted by the activities of transcoding among different register of*	***Expression Communication*** Use multiple media for communication Use multiple tools for construction and composition Build fluencies with graduated levels of support for practice and performance To use different registers in order to communicate

of geometrical properties such as the relations between triangles (Congruent triangles criteria), requires sustained attention and effort. For instance, drawing triangles, visualizing geometrical elements (sides, angles), their relations and regularities require effort persistent and articulated. GeoGebra is an effective tool that supports students in managing drawing, visualization, retrieve geometrical properties at the same time.	*representation (graphical, symbolic)* Support decoding of text, math notation and symbols *This is promoted by the visualization of different registers at the same time*	
Self Regulation Promote expectations and beliefs that optimize motivation Facilitate personal coping skills and strategies Develop self-assessment and reflection	***Comprehension*** Activate or supply background knowledge Highlight patterns, critical features, big ideas, and relationships (checkpoint 3.2) Guide information processing and visualization Maximize transfer and generalization Perception, language and symbols, comprehension (Constructing useable knowledge, knowledge that is accessible for future decision-making, depends not upon merely perceiving information, but upon active "information processing skills") *Visualization associated to the drag function, promotes both perceptual apprehension of figures as well as operative apprehension*	***Executive functions*** *Guide appropriate goal-setting* *The use of visual register of representation to visualize hypotheses on the drawing may be a support for memory and a support for reasoning. This could guide students' process of retrieval of theorem and process of information* *Support planning and strategy development through the ability to connect configuration(s) with geometric principles,* *Facilitate managing information and resources* *Enhance capacity for monitoring progress*

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4. References

1) Duval, R.: 1995a, 'Geometrical Pictures: Kinds of representation and specific processing', in R. Suttherland and J. Mason (eds.), Exploiting Mental Imagery with Computers in Mathematics Education, Springer, Berlin, pp. 142–157.
2) Duval, R.: 1998b, 'Geometry from a cognitive point a view', in C. Mammana and V. Villani (eds.), Perspectives on the Teaching of Geometry for the 21st Century, Kluwer Academic Publishers, Dordrecht, pp. 37–52.
3) Karagiannakis, G. N., Baccaglini-Frank, A. E., & Roussos, P. (2016). Detecting strengths and weaknesses in learning mathematics through a model classifying mathematical skills. Australian J. of Learning Difficulties, 21(2), 115–141.
4) UDL Principles: http://udlguidelines.cast.org/

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