Project Number: 2018-1-IT02-KA201-048274

INTERVENTION TOOL

Understanding the roles played by letters and numbers in Algebra.

1. Introduction

The intervention tool here proposed is aimed to lead students towards an understanding of the roles played by letters and numbers in Algebra, starting from the detection of the difficulties related to the construction of the meaning of variable, of expression depending on such a variable and, in particular, on the different action the position of numbers and letters play in the expression, i.e. on the role of letter and numbers in the expression. In order to develop this educational path we refer to some theoretical frameworks that will be described in the session 2.

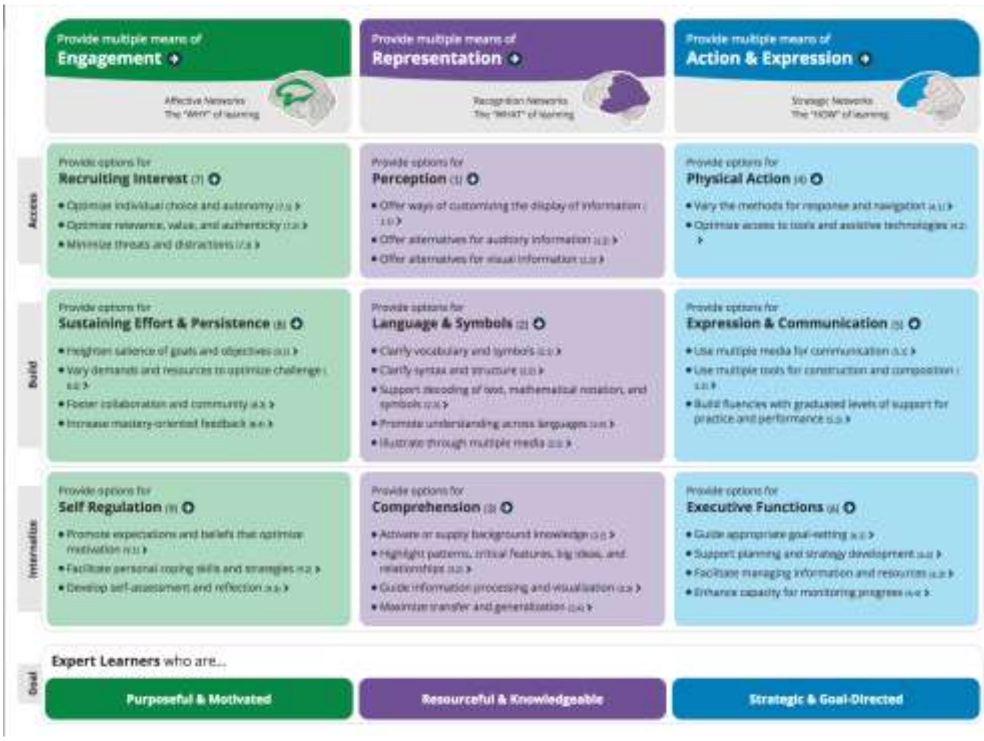
In session 3 the design of the educational activities is described. In particular, if the activities are addressed to a student or the class, the educational aim of the activities, the Cognitive area and math domain of interest and the Mathematical objects in the areas of difficulties identified through the B2 questionnaire

2. Theoretical framework of reference

The theoretical references that helped us to design the following activities are:

1) Universal design for learning (UDL) principles (Table 3), a framework specifically conceived to design *inclusive* educational activities (http://udlguidelines.cast.org/)

Table 3: UDL guidelines



	Provide multiple means of **Engagement** — Affective Networks: The "WHY" of learning	Provide multiple means of **Representation** — Recognition Networks: The "WHAT" of learning	Provide multiple means of **Action & Expression** — Strategic Networks: The "HOW" of learning
Access	Provide options for **Recruiting Interest** (7) • Optimize individual choice and autonomy (7.1) • Optimize relevance, value, and authenticity (7.2) • Minimize threats and distractions (7.3)	Provide options for **Perception** (1) • Offer ways of customizing the display of information (1.1) • Offer alternatives for auditory information (1.2) • Offer alternatives for visual information (1.3)	Provide options for **Physical Action** (4) • Vary the methods for response and navigation (4.1) • Optimize access to tools and assistive technologies (4.2)
Build	Provide options for **Sustaining Effort & Persistence** (8) • Heighten salience of goals and objectives (8.1) • Vary demands and resources to optimize challenge (8.2) • Foster collaboration and community (8.3) • Increase mastery-oriented feedback (8.4)	Provide options for **Language & Symbols** (2) • Clarify vocabulary and symbols (2.1) • Clarify syntax and structure (2.2) • Support decoding of text, mathematical notation, and symbols (2.3) • Promote understanding across languages (2.4) • Illustrate through multiple media (2.5)	Provide options for **Expression & Communication** (5) • Use multiple media for communication (5.1) • Use multiple tools for construction and composition (5.2) • Build fluencies with graduated levels of support for practice and performance (5.3)
Internalize	Provide options for **Self Regulation** (9) • Promote expectations and beliefs that optimize motivation (9.1) • Facilitate personal coping skills and strategies (9.2) • Develop self-assessment and reflection (9.3)	Provide options for **Comprehension** (3) • Activate or supply background knowledge (3.1) • Highlight patterns, critical features, big ideas, and relationships (3.2) • Guide information processing and visualization (3.3) • Maximize transfer and generalization (3.4)	Provide options for **Executive Functions** (6) • Guide appropriate goal-setting (6.1) • Support planning and strategy development (6.2) • Facilitate managing information and resources (6.3) • Enhance capacity for monitoring progress (6.4)
Goal	**Expert Learners** who are... Purposeful & Motivated	Resourceful & Knowledgeable	Strategic & Goal-Directed

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The Center for Applied Special Technology (CAST) has developed a comprehensive framework around the concept of Universal Design for Learning (UDL), with the aim of focusing research, development, and educational practice on understanding diversity and facilitating learning (Edyburn, 2005). UDL includes a set of Principles, articulated in *Guidelines and Checkpoints*[1]. The research grounding UDL's framework is that "learners are highly variable in their response to instruction. [...]"
Thus, UDL focus on these individual differences as an important element to understanding and designing effective instruction for learning.
To this aim, UDL advances three foundational Principles: 1) provide multiple means of representation, 2) provide multiple means of action and expression, 3) provide multiple means of engagement. In particular, guidelines within the first principle have to do with means of perception involved in receiving certain information, and of "comprehension" of the information received. Instead, the guidelines within the second principle take into account the elaboration of information/ideas and their expression. Finally, the guidelines within the third principle deal with the domain of "affect" and "motivation", also essential in any educational activity.
For our analyses we will focus in particular on specific guidelines within the three Principles[2].
Guidelines within Principle 1 (provide multiple means of representation), suggest proposing different options for perception and offering support for decoding mathematical notation and symbols. Moreover, guidelines suggest the importance of providing options for comprehension highlighting patterns, critical features, big ideas, and relationships among mathematical notions. Finally, our analyses will give examples of how software Geogebra[3] can guide visualization and manipulation, in order to maximize transfer and generalization.
Moreover, the guidelines from Principle 2 (provide multiple means of action and expression) suggest to offer different options for expression and communication supporting planning and strategy development. Finally, the guidelines from Principle 3 show how certain activities can recruit students' interest, optimizing individual choice and autonomy, and minimizing threats and distractions.
In the section 4 we will analyse examples of activities, classifying them both by the type of mathematical learning they are designed and the cognitive area they support. We will show how these examples have been designed on the UDL principles in order to make them inclusive and effective to overcame math difficulties identified through B2 questionnaire.

2) The European Project **FasMed** that focused on formative assessment in mathematics and science, (https://research.ncl.ac.uk/fasmed/).
Formative assessment (FA) is conceived as a method of teaching where "evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited" (Black & Wiliam, 2009, p. 7). FaSMEd project refers to Wiliam and Thompson (2007)'s study, that identifies five key strategies for FA practices in school setting: (a) *clarifying and sharing learning intentions and criteria for success;* (b) *engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding;* (c) *providing feedback that moves learners forward;* (d) *activating students as instructional resources for one another;* (e) *activating students as the owners of their own learning.* The teacher, student's peers and the student him- or herself are the agents that activate these FA strategies.

[1] For a complete list of the principles, guidelines and checkpoints and a more extensive description of CAST's activities, visit http://www.udlcenter.org
[2] The items are taken from the interactive list at http://www.udlcenter.org/research/researchevidence
[3] https://wiki.geogebra.org/en/Manual for details.

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Table 4: Formative assessment strategies

	Where the learner is going	Where the learner is right now	How to get there
Teacher	1 Clarifying learning intentions and criteria for success	2 Engineering effective class-room discussions and other learning tasks that elicit evidence of student understanding	3 Providing feedback that moves learners forward
Peer	Understanding and sharing learning intentions and criteria for success	4 Activating students as instructional resources for one another	
Learner	Understanding learning intentions and criteria for success	5 Activating students as the owners of their own learning	

FaSMEd activities are organized in sequences, that encompass group work on worksheets and class discussion where selected group works are discussed by the whole class, under the orchestration of the teacher. Taking into account formative assessment strategies and technology functionalities, Cusi, Morselli & Sabena (2017, p. 758) designed three types of worksheets for the classroom activity:

> “(1) ***Problem worksheets****:* worksheets introducing a problem and asking one or more questions involving the interpretation or the construction of the representation (verbal, symbolic, graphic, and tabular) of the mathematical relation between two variables (e.g. interpreting a time-distance graph);
>
> (2) ***Helping worksheets***: aimed at supporting students who face difficulties with *the problem worksheets* by making specific suggestions (e.g. guiding questions);
>
> (3) ***Poll worksheets***: worksheets prompting a poll among proposed options”.

The authors identified feedback strategies (Table 5) the teacher may adopt to give feedback to students (Cusi, Morselli & Sabena, 2018, p. 3466). These strategies are employed in the class discussion that is organized by the teacher after the group work on worksheets.

Table 5:

Re-voicing	When the teacher mirrors one student’s intervention so as to draw the attention on it. Often, during the re-voicing, the teacher stresses with voice intonation some crucial words of the sentence she is mirroring. Rephrasing takes place when the teacher reformulates the intervention of one student, with the double aim of drawing the attention of the class and making the intervention more intelligible to everybody.
Rephrasing	Rephrasing takes place when the teacher reformulates the intervention of one student, with the double aim of drawing the attention of the class and making the intervention more intelligible to everybody. Rephrasing is applied when the teacher feels that the intervention could be useful but needs to be communicated in a better way so as to become a resource for the others. […] The revoicing and rephrasing strategies […] turn one student (the author of the intervention) into a resource for the class.
Rephrasing with scaffolding	When the teacher, besides rephrasing, adds some elements to guide the students’ work.
Relaunching	When the teacher reacts to a student’s intervention, which (s)he considers interesting for the class, not giving a direct feedback, but posing a connected question. In this way, by relaunching the teacher provides an implicit feedback […] on the student’s intervention, suggesting that the issue is interesting and worth to be deepened or, conversely, has some problematic points and should be reworked on.

Contrasting	Contrasting takes place when the teacher draws the attention on two or more interventions, representing two different positions, so as to promote a comparison. By contrasting, […] the authors of the two positions may be resource for the class as well as responsible of their own learning.

We draw from the FaSMEd experience the idea of creating classroom activities in the formative assessment perspective, which may promote inclusion.

3. Design

3.1 Difficulties identified through the B2 questionnaire

We detect difficulties in the following item of B2 questionnaire (Q4AI4):

If x=2, complete the following expressions: $x^2 = \dots$
$2x = \dots$
$x2 = \dots$

These difficulties are related to the construction of the meaning of variable, of expression depending on such a variable and, in particular, on the different action the position of numbers and letters plays in the expression, i.e. on the role of letters and numbers in the expression.

3.2 Cognitive area and math domain of interest

The area of difficulties identified through the B2 questionnaire is related to the domain of *Algebra*. In particular, the difficulties are related to the construction of the meaning of variable, of expression depending on such a variable and, in particular, on the different action the position of numbers and letters play in the expression, i.e. on the role of letter and numbers in the expression. Thus, *Visuospatial* is the cognitive area involved (Table 1).

Table 1: The difficulties detected are linked to the cognitive domain of *Visuospatial* and in the domain of *Algebra*

	Arithmetic	Geometry	Algebra
Memory			
Reasoning			
Visuospatial			If x=2, complete the following expressions: $x^2 = \dots$ $2x = \dots$ $x2 = \dots$

3.3 Educational Aims

The intervention tool is aimed to lead students towards an *Understanding the roles played by letters and numbers in Algebra.*

3.4 Addressing to Student /class

The Intervention tool is articulated in a set of activities that has to be carried out with all the class, in a perspective of inclusion.

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3.5 Educational activities: the Intervention Tool

The teaching sequences are conceived to address specific learning difficulty, within an inclusive perspective. The activities are, first of all, thought to empathize with students concerning their difficulties approaching algebra. According to Villani (2014) there is also a physiological contribute for students in moving towards algebra. Most of teacher and books state that: "In Algebra we act with letters as we act with numbers", but it is really true? Starting from this food for thoughts we can try to share with students their emerging doubts related to the proposed exercise, analysing the semantic aspect behind the three given expressions. Then, to clarify into a visuospatial frame the meaning and so the similarities and differences of the calculations varying coefficients and variables, ICT tools are used, in particular Geogebra, a Dynamic Geometry software.

Food for thoughts on approaching algebra.

The first step is then to share with students their difficulties approaching algebra. We can open a discussion among the class around the statement: "In Algebra we act with letters as we act with numbers". We could adopt the debate methodology: we divide class into two sides. Naturally, one will argue for and another against the resolution stated. The best choice to make the methodology work, is to break the class into four groups and assign two groups to each of two resolutions. Then we assign one of each pair of student groups to the affirmative. This group will argue for the issues being presented. The other two groups will be the negative and will argue against the resolutions. During the debate, the other groups will serve as the *judges* and decide which side presented a stronger case voting for the *winners* of the debate at its *conclusion*.

The teacher acts just as a supervisor, keeping track of the timing of the different phases of the debate, giving students additional instruction on the specific vocabulary that may be involved and examples to use to carry on their thesis. For example we can give the following expressions: $\frac{a+b}{a} = \frac{\cancel{a}+b}{\cancel{a}} = b$; $\frac{a}{ab} = \frac{\cancel{a}}{\cancel{a}b} = \frac{0}{b} = 0$; $(a+b)^2 = a^2 + b^2$ asking students to debate why are the most common mistakes done by students all over the world!
We can also use the same expressions but using numbers asking students to perform the operations in order to make a comparison between arithmetic and algebra.
With this first speech activity, also using written notes of the debate, we can experiment the five feedback strategies of FeSMEd recalled in Table 5.

We can, in particular, draw a tracking shot of the different terms used by students, stressing on the meaning and role of letters and numbers in an expression, for instance the difference between variable and unknown among letters and between coefficient and exponent among numbers. Furthermore, we can discuss the difference between a result in arithmetic and in algebra. In particular, it is possible that among the arguments against the statement that in algebra we act as in arithmetic, some students has noticed that in algebra we often obtain a result that is a further algebraic expression while among operations with numbers we obtain just a number and that this is one of the motivation of several mistakes in algebra, like the ones given as examples above.

These mistakes are very common among students, since they are psychologically compelled to reach just a letter as a result like with numbers. If the same expressions are written with numbers, students would do the calculation correctly since with numbers they are able to control semantically the procedure while with letters this control is loosed, all the certainties to something of familiar and concrete is missed.
This kind of deepening even through different expressions compared to the one we have to solve, has the aim of making students think of the differences and similarities between algebra and arithmetic heartening them respect to the fact that it is in some way normal to be at the beginning dazed passing from calculation among numbers to calculation with letters.

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First step: mathematical interpretation of the given expressions

As a first approach to better understand the different meaning of the expressions, we propose to open a discussion in the class on the significance of the operation to perform.

This activity will be done among positive integer's numbers.

Students will be invited to write a definition by their own words, and then the teacher will collect the worksheets and will just moderate the discussion, according to the 5 principles collected in table 5.

Hopefully, we will reach a shared and correct definition of multiplication as a different way of doing an addition, a more compact way of writing an addition, i.e. *for positive integers,* ***multiplication*** *consists of adding a number (the multiplicand) to itself a specified number of times given by another number (the multiplier). The result is called product.*

We can start with x^2 when $x = 3$ instead of 2 to look at $x = 2$ as a special case at the end of the procedure. We have also to specify that x^2, in turn, is a compact way of writing a multiplication of the number by itself.
So $x^2 = 3^2 = 3 \cdot 3 = 3 + 3 + 3 = 9$ while$2x = 2 \cdot 3 = 3 + 3 = 6$.
We then ask students to iterate the procedure with different values of *x* in order to get convinced of the difference between the two operations even if both can be connected to a sum, i.e. we can begin to look at the different role played by 2 according to its position respect to the letter in the expression.
By this strategy, we can deeply explore the real meaning of the two expressions. Recalling then the commutative principle for the addition we can show that $x2 = 2x$ and so they give the same result, trying again with specific numbers in place of *x.*

After that we can then focus on the fact that in case of $x = 2$ the equal results (4) from the different operations is just a mere accident and that normally the two expressions means different operation to be performed and, generally speaking, the results are then different.

The visuospatial frame: the geometrical viewpoint
After this preliminary step conducted to get familiar with the problem to solve, we can switch to the core of the class activity involving the visuospatial approach and the geometrical viewpoint, in particular, to move the previous activity down to a specific concrete area of interest.

By Geogebra we draw a segment of length 2 that we call *x*. Then we can construct a square of side *x=2* to have a geometrical representation of the expression $x^2 = 2^2 = 4$ as the area of the square (Fig. 1).

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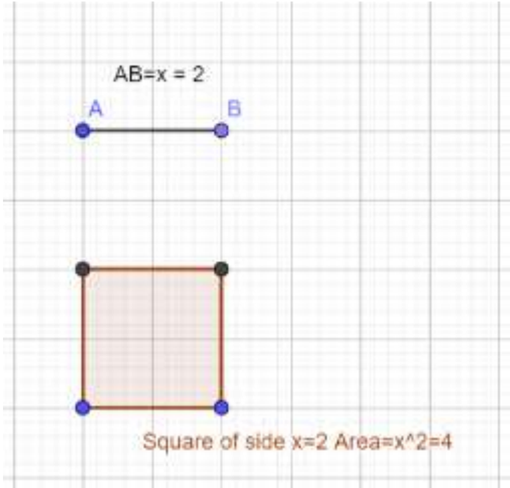

Fig. 1 *Construction of a square of side 2 starting from a segment of length x=2 (Geogebra)*

After that we can ask student to represent x2 and 2x on their notebooks. Depending on their answers, a discussion is open.
We can also do it by the software Geogebra (Fig.2).

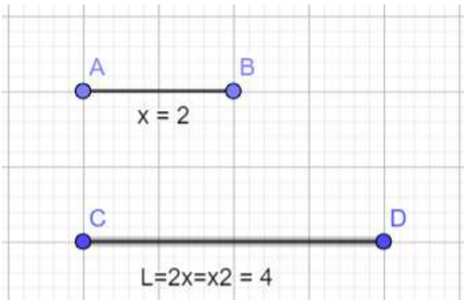

Fig. 2 *Construction of a segment that has double length of segment AB (Geogebra)*

If it is clear to students that x2 is the same of 2x and that it is different from x^2 because they will correctly draw a segment of double length compared to x, we can ask them to explain each other the different role played by the number 2 if placed as exponent or as factor.
Even if in the case of x=2 the mathematical result is the same, by means of the visual representation of geometrical elements, it will be evident at a glance that in the first case we get an area while in the latter still a segment.
This consideration will be obviously stressed especially in the case of a wrong answer from students.
We can also make a reflection on units of measure and dimensions of the quantity involved, recalling also basic physical concepts related to measurements.
We can then put the focus on what will happen if we change the length of the segment, e.g. setting at 3 the value of x. This could be easily done in Geogebra setting *x* as a variable and by this way we can further discuss the concept of variable, the role of the letters at stake in the different expressions used and emphasize the power of algebra versus arithmetic.
This is achieved by sliding the value of the length of the segment associated with *x* and the correspondent changing in the result of 2x as shown in fig 2b (during the animation students can follow the tracking point).

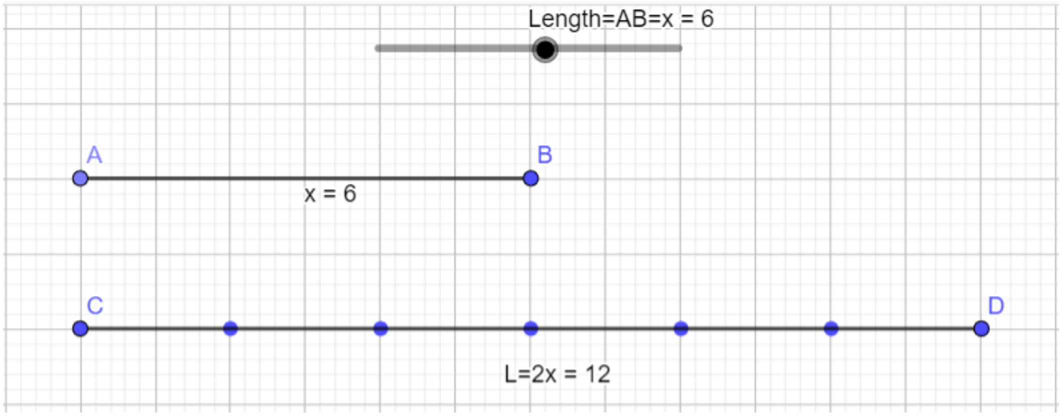

Fig. 2b *Construction of a segment that has double length of segment AB sliding the value of the variable x (Geogebra)*

Representation of the relation between variable and expression depending on such a variable on a Cartesian plan and on a table

Students themselves will then be asked to try to give a definition in order to conceptualize the meaning of variable versus parameter.
It could be done again by a visual approach, tracking the two functions $y = 2x$ and $= x^2$. At first, we can ask them to plot by points the graph of the two functions on their own notebook, and then we can do it with Geogebra.
We consider a table defining the relation between the variable “x” and the expression 2x.

x	**2x**	**x^2**
0		
1		
2		
3		

The teacher asks to the students to calculate the value of the expression 2x starting from the values of the independent variable “x”

x	**2x**	**x^2**
0	2·0=0	0·0=0
1	2·1=2	1·1=1
2	2·2=4	2·2=4
3	2·3=6	3·3=9

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The teacher asks students to draw the relation on the Cartesian plane:

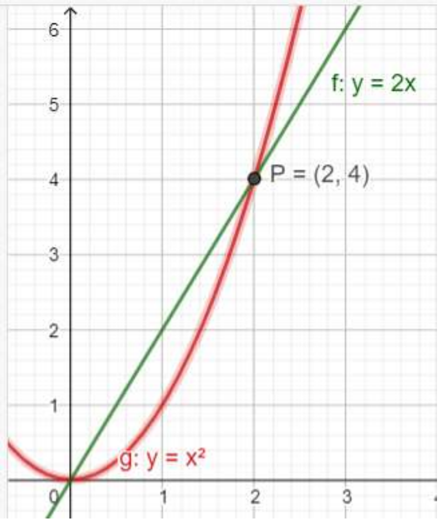

Fig. 3 *Plot of* $y = 2x$ *and of* $y = x^2$ *(Geogebra)*

The teacher guides the discussion about the relation between *x* and the expressions *2x* and x^2 both through geometrical representation (on the Cartesian plan) and the algebraic relation (on the table) so that students will be able to pass from a code to the other one (transcoding process).
At this stage we could also ask students to focus on the intersection point to observe when it occurs.

A step further could be to track $y = kx$ varying the value of *k* in Geogebra and opening discussion on the role *k* plays compared to x.

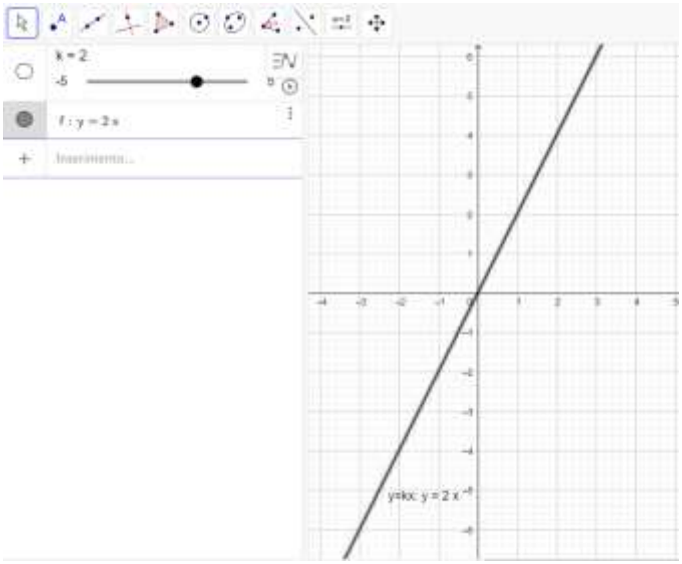

Fig. 4 *Plot of* $y = kx$ *in Geogebra*

The teacher can ask: “What do you think will happen to the graph changing *k”*?
After a discussion and some trials performed by students on their notebook, using tables and plotting by points their results, the teacher will show what happens changing the value of *k* moving the dot on the line of k in *Geogebra* (Fig. 5)

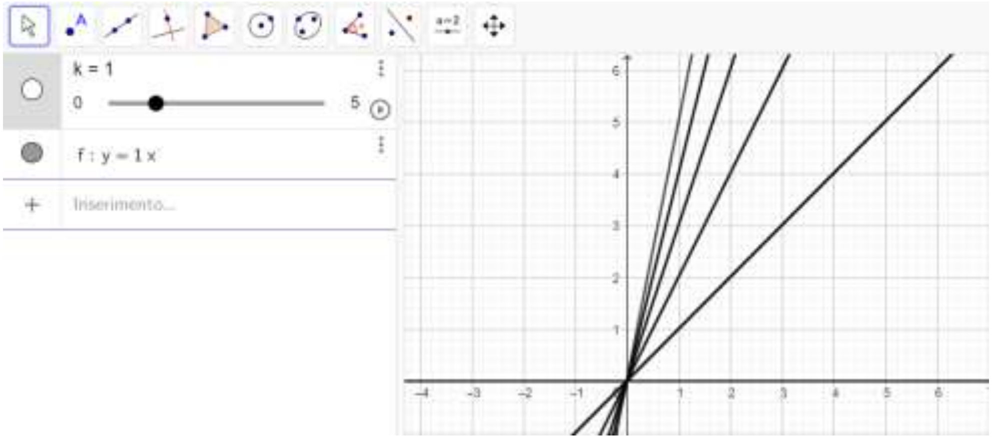

Fig. 5 *Plot of* $y = kx$ *in Geogebra* obtained varying *k* and contemporary tracking the plot.

“How do you interpret what happens to the algebraic expression 2*x, 3x and so on*?”.

We can ask students to go back to the worksheet involving segments to point out how *k* affects the result and what kind of mathematical relationship exists between the final length obtained multiplying *x* by 2, 3 etc. and *x* itself.

A final and more in depth debate could be open aimed to give meaning to letters in the expressions in spite of the fact that equal letters are often used, proposing to discuss the different meaning of *x* in the two following expressions: $lenght = 2x$ and $2x = 4$, identifying in the first *x* as a variable (e.g. the length of the different segments to be doubled) and in second *x* as an unknown value we’d like to determine to know what length has to be associated to a segment to make its double being 4 and so on (or whatever quantity is associated with *x*).

We note that the described approaches propose different representations (UDL Principle 1) and they are thought to play as mediator of the algebraic concepts of *variable*, *dependent expression* and of the role of numbers in them (*numbers as factors* or *exponents*) through a dynamic model ranging from mathematical to geometrical to graphical point of view. (UDL Principle 2). The mediation can occur thanks to visual channel and using visual verbal means (written language) just downstream of a visualization of the sense of that means. The construction of the concept realized as so may allow students, and especially students with MLD, to find experiential references that fit their cognitive style, providing multiple means of engagement (UDL Principle 3).
This will lead them to retrieve this approach in their memory each time they will face an algebraic expression involving similar situations, making them more confident to succeed.

In terms of formative assessment, our design activates strategy 2 (engineering classroom discussions). During the discussion, are instead experimented strategies 5 and 4, since students may express their doubts becoming owners of their own learning or give explanations to their mates becoming resources for the mates. The teacher and the peers may provide feedback to a student, thus activating strategy 3.

4. Discussion through UDL guidelines about the above-mentioned activities

We observe that the same educational aim of *Constructing an understanding of the various roles played by letters and numbers in Algebra* is approached in different ways by acting on the three principles of UDL (Table 7, in *red* our comments to illustrate the connection between the principles and our activities).

Table 7: Analysis of the activities through the Table of UDL principles.

Engagement	***Representation***	***Action & Expression***
Recruiting interest Optimize individual choice and autonomy Optimize relevance, value, and authenticity Minimize threats and distractions	***Perception*** Offer ways of customizing the display of information Offer alternatives for auditory information Offer alternatives for visual information Different registers through which information are displayed (visual; visual-dynamic; symbolic)	***Physical Action*** Vary the methods for response and navigation Optimize access to tools and assistive technologies
Sustaining effort Persistence Heighten salience of goals and objectives Vary demands and resources to optimize challenge Foster collaboration and community Increase mastery-oriented feedback Vary demands and resources to optimize challenge Foster collaboration and community Oriented feedbacks support engagement and motivation with respect the elaboration of the solution of the task	***Language & Symbols*** Clarify vocabulary and symbols Clarify syntax and structure Offer alternative language and symbols to decode information and to work on the information Support decoding of text, mathematical notation, and symbols *This is promoted by the dynamic action due to use of the Dynamic Geometry software.* Promote understanding across languages Illustrate through multiple media *This is promoted by the activities of transcoding among different registers of representation* Support decoding of text, math notation and symbols *This is promoted by the visualization of different registers (for example, geometrical interpretation and visualization, relationships among mathematical operations and different role played by same numbers and letters; a variable as a mobile point labelled by x*	***Expression Communication*** Use multiple media for communication Use multiple tools for construction and composition Build fluencies with graduated levels of support for practice and performance To use different registers in order to communicate *In the activities mathematical manipulatives are provided. For instance, dragging a moving point may help visualizing that the parameter may have different values affecting the graph in the plot or variable may have different values affecting the corresponding result of the expression.*

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	that is a length)	
Self-Regulation Promote expectations and beliefs that optimize motivation Facilitate personal coping skills and strategies Develop self-assessment and reflection *Formative assessment strategies, as discussed in section 2, may help self-assessment and reflection. More specifically, the teacher may provide different types of feedback*	***Comprehension*** Activate or supply background knowledge Highlight patterns, critical features, big ideas, and relationships (checkpoint 3.2) Guide information processing and visualization Maximize transfer and generalization Perception, language and symbols, comprehension (Constructing useable knowledge, knowledge that is accessible for future decision-making, depends not upon merely perceiving information, but upon active "information processing skills")	***Executive functions*** *Guide appropriate goal-setting* *The use of artefacts may also be a support for memory. Artefacts guide students' process of inquiry, providing feedback to their process. (for example through the geometrical visualization of the algebraic expressions)* *Support planning and strategy development* *Facilitate managing information and resources*

This allows students to construct meaning for the algebraic notions at stake.

5. References

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