Project Number: 2018-1-IT02-KA201-048274

INTERVENTION TOOL

Comparing decimal numbers

1. Introduction

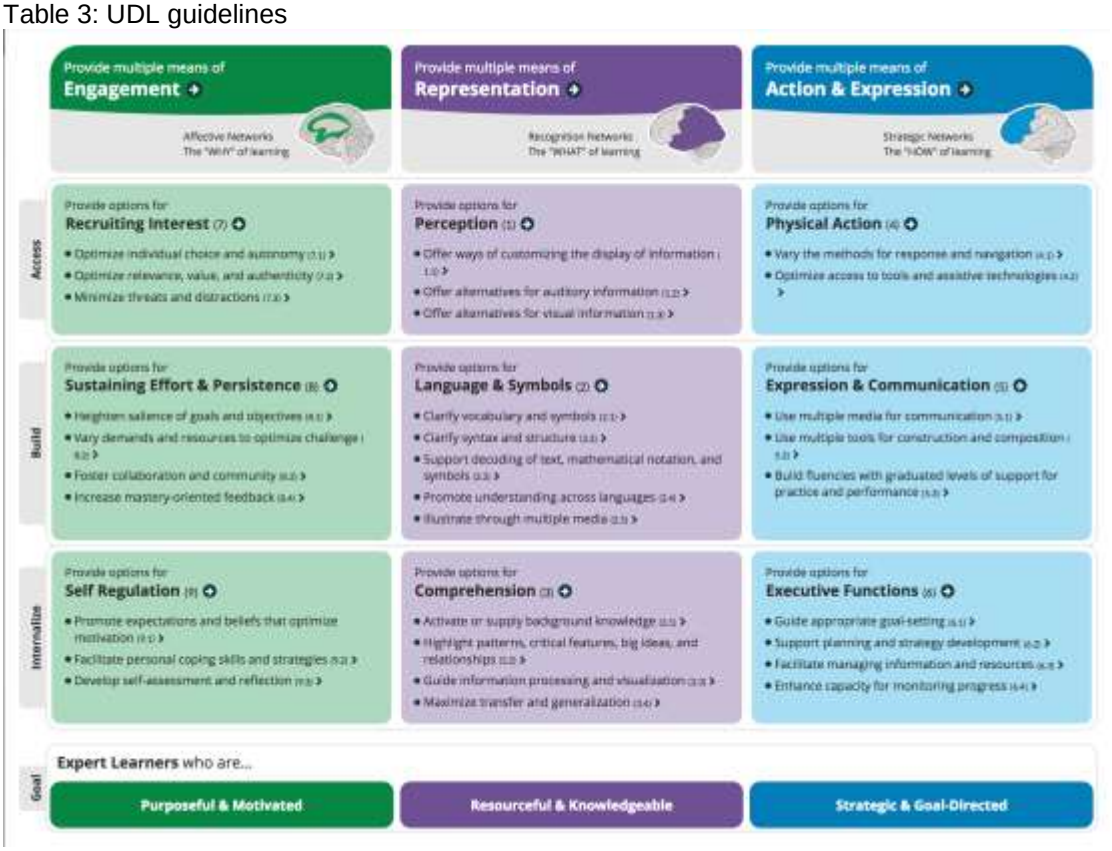
In order to develop a set of educational activities aimed to compare decimal numbers, we refer to some theoretical frameworks that will be described in the session 2.
In session 3 the design of the educational activities is described. In particular, if the activities are addressed to students or the class, the educational aim of the activities, the Cognitive area and math domain of interest and the Mathematical objects in the areas of difficulties identified through the B2 questionnaire

2. Theoretical framework of reference

The theoretical references that helped us to design the following activities are:

1) **Universal design for learning (UDL) principles** (Table 3), a framework specifically conceived to design *inclusive* educational activities (http://udlguidelines.cast.org/)

Table 3: UDL guidelines



The Center for Applied Special Technology (CAST) has developed a comprehensive framework around the concept of Universal Design for Learning (UDL), with the aim of focusing research, development, and educational practice on understanding diversity and facilitating learning (Edyburn, 2005). UDL includes a set of Principles, articulated in

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Guidelines and Checkpoints[1]. The research grounding UDL's framework is that "learners are highly variable in their response to instruction. [...]"
Thus, UDL focus on these individual differences as an important element to understanding and designing effective instruction for learning.
To this aim, UDL advances three foundational Principles: 1) provide multiple means of representation, 2) provide multiple means of action and expression, 3) provide multiple means of engagement. In particular, guidelines within the first principle have to do with means of perception involved in receiving certain information, and of "comprehension" of the information received. Instead, the guidelines within the second principle take into account the elaboration of information/ideas and their expression. Finally, the guidelines within the third principle deal with the domain of "affect" and "motivation", also essential in any educational activity.
For our analyses we will focus in particular on specific guidelines within the three Principles[2].
Guidelines within Principle 1 (provide multiple means of representation), suggest proposing different options for perception and offering support for decoding mathematical notation and symbols. Moreover, guidelines suggest the importance of providing options for comprehension highlighting patterns, critical features, big ideas, and relationships among mathematical notions. Finally, our analyses will give examples of how software AlNuSet can guide information processing, visualization, and manipulation, in order to maximize transfer and generalization.
Moreover, the guidelines from Principle 2 (provide multiple means of action and expression) suggest to offer different options for expression and communication supporting planning and strategy development. Finally, the guidelines from Principle 3 show how certain activities can recruit students' interest, optimizing individual choice and autonomy, and minimizing threats and distractions.
In the section 4 we will analyse examples of activities, classifying them both by the type of mathematical learning they are designed and the cognitive area they support. We will show how these examples have been designed on the UDL principles in order to make them inclusive and effective to overcame math difficulties identified through B2 questionnaire.

2) The European Project **FasMed,** that focused on formative assessment in mathematics and science, (https://research.ncl.ac.uk/fasmed/).
Formative assessment (FA) is conceived as a method of teaching where "evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited" (Black & Wiliam, 2009, p. 7). FaSMEd project refers to Wiliam and Thompson (2007)'s study, that identifies five key strategies for FA practices in school setting: (a) *clarifying and sharing learning intentions and criteria for success;* (b) *engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding;* (c) *providing feedback that moves learners forward;* (d) *activating students as instructional resources for one another;*–(e) *activating students as the owners of their own learning.* The teacher, student's peers and the student him- or herself are the agents that activate these FA strategies.

[1] For a complete list of the principles, guidelines and checkpoints and a more extensive description of CAST's activities, visit http://www.udlcenter.org
[2] The items are taken from the interactive list at http://www.udlcenter.org/research/researchevidence

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Table 4: Formative assessment strategies

	Where the learner is going	Where the learner is right now	How to get there
Teacher	**1** Clarifying learning intentions and criteria for success	**2** Engineering effective class-room discussions and other learning tasks that elicit evidence of student understanding	**3** Providing feedback that moves learners forward
Peer	Understanding and sharing learning intentions and criteria for success	**4** Activating students as instructional resources for one another	
Learner	Understanding learning intentions and criteria for success	**5** Activating students as the owners of their own learning	

FaSMEd activities are organized in sequences, that encompass group work on worksheets and class discussion where selected group works are discussed by the whole class, under the orchestration of the teacher. Taking into account formative assessment strategies and technology functionalities, Cusi, Morselli & Sabena (2017, p. 758) designed three types of worksheets for the classroom activity:

> "(1) *problem worksheets:* worksheets introducing a problem and asking one or more questions involving the interpretation or the construction of the representation (verbal, symbolic, graphic, tabular) of the mathematical relation between two variables (e.g. interpreting a time-distance graph);
>
> (2) *helping worksheets*, aimed at supporting students who face difficulties with *the problem worksheets* by making specific suggestions (e.g. guiding questions);
>
> (3) *poll worksheets*: worksheets prompting a poll among proposed options".

The authors identified feedback strategies (Table 5) the teacher may adopt to give feedback to students (Cusi, Morselli & Sabena, 2018, p. 3466). These strategies are employed in the class discussion that is organized by the teacher after the group work on worksheets.

Table 5:

Revoicing	When the teacher mirrors one student's intervention so as to draw the attention on it. Often, during the revoicing, the teacher stresses with voice intonation some crucial words of the sentence she is mirroring. Rephrasing takes place when the teacher reformulates the intervention of one student, with the double aim of drawing the attention of the class and making the intervention more intelligible to everybody.
Rephrasing	Rephrasing takes place when the teacher reformulates the intervention of one student, with the double aim of drawing the attention of the class and making the intervention more intelligible to everybody. Rephrasing is applied when the teacher feels that the intervention could be useful but needs to be communicated in a better way so as to become a resource for the others. […] The revoicing and rephrasing strategies […] turn one student (the author of the intervention) into a resource for the class.
Rephrasing with scaffolding	When the teacher, besides rephrasing, adds some elements to guide the students' work.
Relaunching	When the teacher reacts to a student's intervention, which (s)he considers interesting for the class, not giving a direct feedback, but posing a connected question. In this way, by relaunching the teacher provides an implicit feedback […] on the student's intervention, suggesting that the issue is interesting and worth to be deepened or,

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	conversely, has some problematic points and should be reworked on.
Contrasting	Contrasting takes place when the teacher draws the attention on two or more interventions, representing two different positions, so as to promote a comparison. By contrasting, […] the authors of the two positions may be resource for the class as well as responsible of their own learning.

We draw from the FaSMEd experience the idea of creating classroom activities in the formative assessment perspective, which may promote inclusion.

3. Design

In the subsections 3.1-3.5, the activities of the intervention tool are presented in detail**.**

3.1 Difficulties identified through the B2 questionnaire

The intervention tool is presented in reference to a specific difficulty that was detected by means of the questionnaire.

We detect difficulties in the following item of B2:
Order the following numbers from the smallest to the greatest:
0.233, 0.3, 0.32, 0.35, 0.208
These difficulties are related to the understanding of the position decimal system.

3.2 Cognitive area and math domain of interest

The specific difficulty that is mentioned in subsection 3.1 is linked to the cognitive area related to Core Number and the mathematical domain of interest is Arithmetic.

3.3 Educational Aims

The intervention tool is aimed at promoting understanding of the position decimal system, and understanding of how to compare decimal numbers.

3.4 Addressing to Student /class

The Intervention tool is articulated in a set of activities that have to be carried out with all the class, in a perspective of inclusion.

3.5 Educational activities: the Intervention Tool

Order the following numbers from the smallest to the greatest:

3.21; 3.20; 2.87

In order to provide the students with a new way of customising the display of information (UDL guidelines) the teacher suggests the following method:

a) Think, which number is the smallest and which is the greatest. Please, place the numbers on a number line (approximately). Numbers on the left are *smaller* than the numbers on the right of the *number line*.

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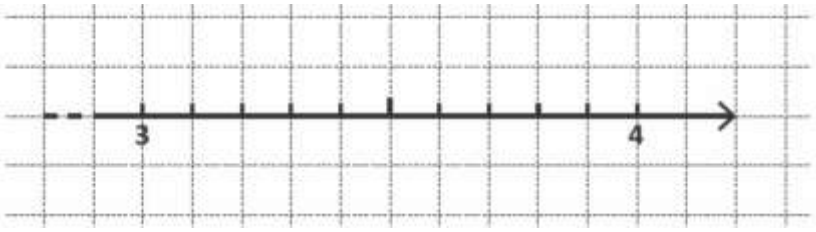
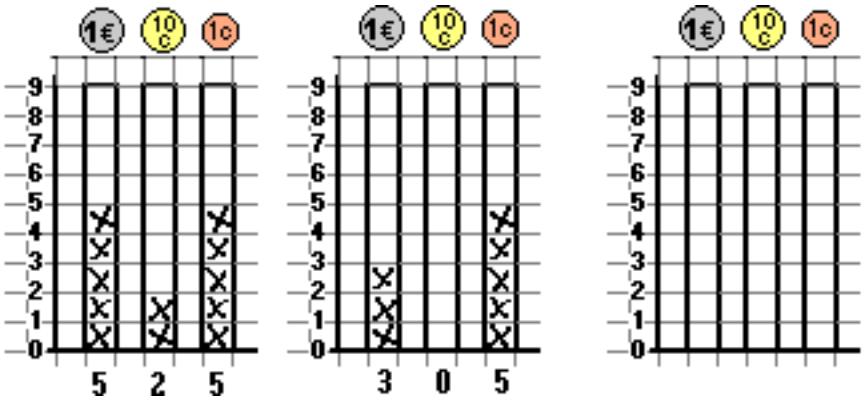


In order to engineer effective classroom discussions (FaSMEd), the teacher initiates a discussion starting from students' answers. Teacher shows the right answer on the table.

b) The teachers can use the money abacus to represent the numbers: 5,25 and 3,05.



The teacher explains how the money abacus works with the given examples of 5,25 and 3,05 and asks students to try to the same with one of the numbers 3.21; 3.20; 2.87. This will again enable the students to have access to multiple means of representation (UDL guidelines).

c) Teacher asks students to fill the table and then compare decimals. Students work in pairs, thanks to which they will be able to cooperate and offer instructional resources for one another (FaSMEd).

	UNITS	TENTHS	HUNDREDTHS
3.21	***	**	*
3.20			
2.87			

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d) The teacher explains how students can compare decimal numbers:

- **3.21**=3+ 0.2 + 0.01

  = 3 units + 2 tenths + 1 hundredths

- **3.20** =3+ 0.2 + 0.00

  = 3 units + 2 tenths + 0 hundredths

- **2.87**=2+ 0.8 + 0.07

  = 2 units + 8 tenths + 7 hundredths + 9 thousandths

Because 2 units is less than 3 units, 2.87 is smaller than 3.21 and 3.20.

Now, compare 3.21 and 3.20.

Because 3 units=3 units you need to compare next digit

2 tenths=2 tenths, as they are the same, you need to compare another digit:

1 hundredths > 0 hundredths thus, 3.20 < 3.21

So, as you need to order the numbers from *the smallest to the greatest:*

2.87 < 3.20 < 3.21

e) Teacher shows another way of comparing decimals:

Now, try to observe the integral part

3.21; 3.20; 2.87

3=3>2
2.87 is the smallest number

When the integral part is same then compare the tenths place

3.21; 3.20

2=2

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When the tenth place is same compare the hundredths place.

3.21; 3.20

1 < 0

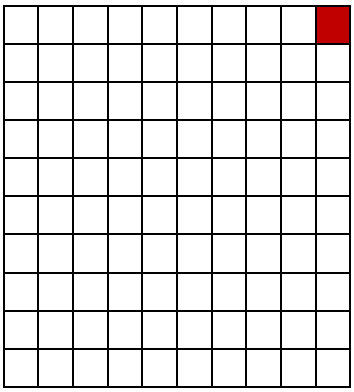
3.21 is the greatest number.

Thanks to this activity the students will have an alternative way for visual information (UDL guidelines)

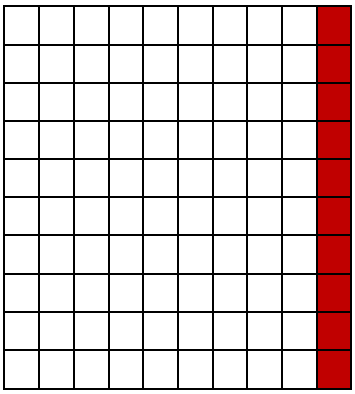
f) Teacher asks students to do this activity using squares:

Try to draw the squares which will represent numbers 3.21; 3.20; 2.87 and then compare them. Remember that:

0.01



0.1



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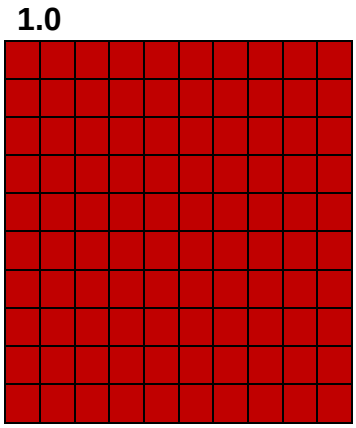

Now, try to draw and colour the squares by yourselves in the way they show representation of 3.21; 3.20; 2.87.
Compare the outcome with your classmate. Now try to write the numbers from the smallest to the greatest.

The students compare their outcomes with others. The teacher checks the answers and provides feedback so that the students can move forward (FaSMEd).

4) References

[1] Karagiannakis, G. N., Baccaglini-Frank, A. E., & Roussos, P. (2016). Detecting strengths and weaknesses in learning mathematics through a model classifying mathematical skills. Australian J. of Learning Difficulties, 21(2), 115–141. https://doi.org/10.1080/19404158.2017.1289963
[2] Workshop with Dr. Giannis Karagianakis in International
[3] European Project FasMed (https://research.ncl.ac.uk/fasmed/).
[4] Universal design for learning (UDL) principles (http://udlguidelines.cast.org/)

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