Project Number: 2018-1-IT02-KA201-048274

INTERVENTION TOOL

Multiplying powers which have the same base

1. Introduction

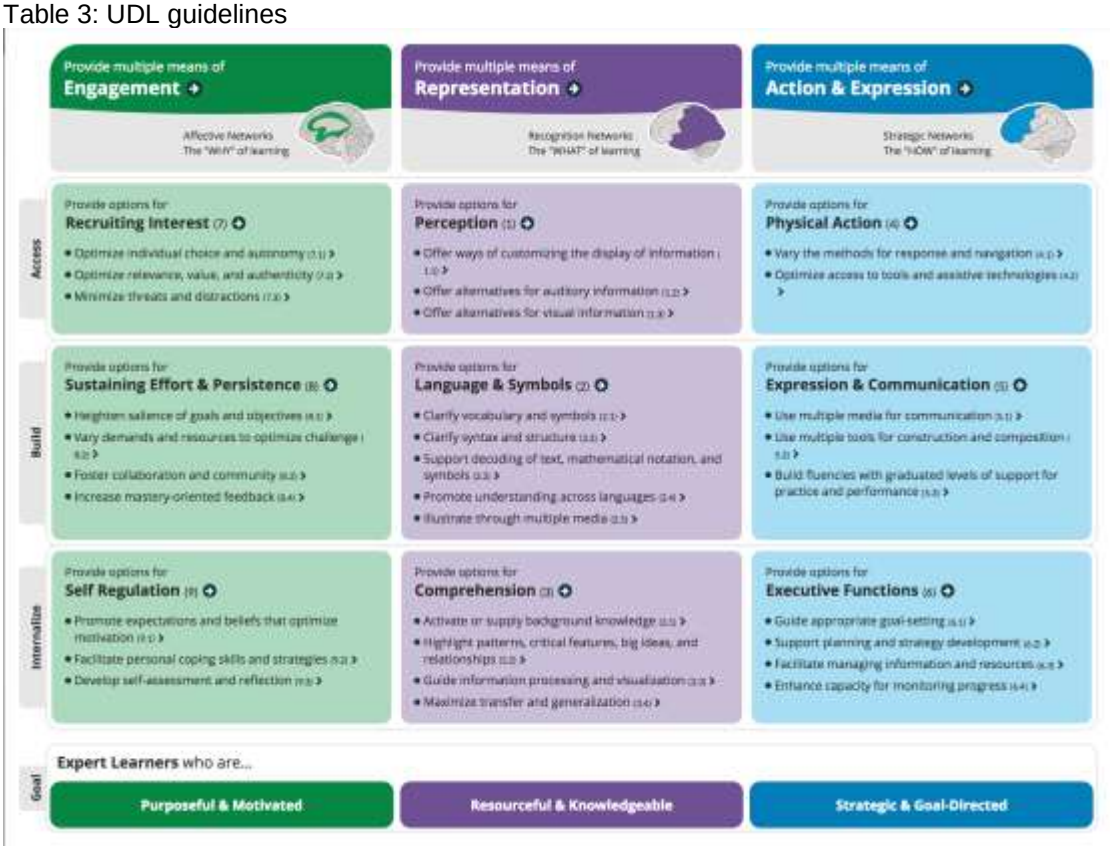
In order to develop a set of educational activities aimed to construct the meaning of power and the multiplication of powers with the same base, we refer to some theoretical frameworks that will be described in the session 2.
In session 3 the design of the educational activities is described. In particular, if the activities are addressed to students or the class, the educational aim of the activities, the Cognitive area and math domain of interest and the Mathematical objects in the areas of difficulties identified through the B2 questionnaire

2. Theoretical framework of reference

The theoretical references that helped us to design the following activities are:

1) **Universal design for learning (UDL) principles** (Table 3), a framework specifically conceived to design *inclusive* educational activities (http://udlguidelines.cast.org/)

Table 3: UDL guidelines



The Center for Applied Special Technology (CAST) has developed a comprehensive framework around the concept of Universal Design for Learning (UDL), with the aim of focusing research, development, and educational practice on understanding diversity and facilitating learning (Edyburn, 2005). UDL includes a set of Principles, articulated in

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Guidelines and Checkpoints[1]. The research grounding UDL's framework is that "learners are highly variable in their response to instruction. [...]"
Thus, UDL focus on these individual differences as an importantfelement to understanding and designing effective instruction for learning.
To this aim, UDL advances three foundational Principles: 1) provide multiple means of representation, 2) provide multiple means of action and expression, 3) provide multiple means of engagement. In particular, guidelines within the first principle have to do with means of perception involved in receiving certain information, and of "comprehension" of the information received. Instead, the guidelines within the second principle take into account the elaboration of information/ideas and their expression. Finally, the guidelines within the third principle deal with the domain of "affect" and "motivation", also essential in any educational activity.
For our analyses we will focus in particular on specific guidelines within the three Principles[2].
Guidelines within Principle 1 (provide multiple means of representation), suggest proposing different options for perception and offering support for decoding mathematical notation and symbols. Moreover, guidelines suggest the importance of providing options for comprehension highlighting patterns, critical features, big ideas, and relationships among mathematical notions. Finally, our analyses will give examples of how software AlNuSet can guide information processing, visualization, and manipulation, in order to maximize transfer and generalization.
Moreover, the guidelines from Principle 2 (provide multiple means of action and expression) suggest to offer different options for expression and communication supporting planning and strategy development. Finally, the guidelines from Principle 3 show how certain activities can recruit students' interest, optimizing individual choice and autonomy, and minimizing threats and distractions.
In the section 4 we will analyse examples of activities, classifying them both by the type of mathematical learning they are designed and the cognitive area they support. We will show how these examples have been designed on the UDL principles in order to make them inclusive and effective to overcame math difficulties identified through B2 questionnaire.

2) The European Project **FasMed,** that focused on formative assessment in mathematics and science, (https://research.ncl.ac.uk/fasmed/).
Formative assessment (FA) is conceived as a method of teaching where "evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited" (Black &Wiliam, 2009, p. 7). FaSMEd project refers to Wiliam and Thompson (2007)'s study, that identifies five key strategies for FA practices in school setting: (a) *clarifying and sharing learning intentions and criteria for success;* (b) *engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding;* (c) *providing feedback that moves learners forward;* (d) *activating students as instructional resources for one another;*(e) *activating students as the owners of their own learning.* The teacher, student's peers and the student him- or herself are the agents that activate these FA strategies.

[1] For a complete list of the principles, guidelines and checkpoints and a more extensive description of CAST's activities, visit http://www.udlcenter.org
[2] The items are taken from the interactive list at http://www.udlcenter.org/research/researchevidence

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Table 4: Formative assessment strategies

<table>
<tr><th></th><th>Where the learner is going</th><th>Where the learner is right now</th><th>How to get there</th></tr>
<tr><td>Teacher</td><td rowspan="3">1 Clarifying learning intentions and criteria for success</td><td>2 Engineering effective class-room discussions and other learning tasks that elicit evidence of student understanding</td><td>3 Providing feedback that moves learners forward</td></tr>
<tr><td>Peer</td><td colspan="2" rowspan="2">4 Activating students as instructional resources for one another</td></tr>
<tr><td>Learner</td></tr>
<tr><td></td><td>Understanding and sharing learning intentions and criteria for success</td><td colspan="2">5 Activating students as the owners of their own learning</td></tr>
<tr><td></td><td>Understanding learning intentions and criteria for success</td><td colspan="2"></td></tr>
</table>

FaSMEd activities are organized in sequences, that encompass group work on worksheets and class discussion where selected group works are discussed by the whole class, under the orchestration of the teacher. Taking into account formative assessment strategies and technology functionalities, Cusi, Morselli& Sabena (2017, p. 758) designed three types of worksheets for the classroom activity:

> "(1) *problem worksheets:* worksheets introducing a problem and asking one or more questionsinvolving the interpretation or the construction of the representation (verbal, symbolic, graphic, tabular) of the mathematical relation between two variables (e.g. interpreting a time-distance graph);
>
> (2) *helping worksheets*, aimed at supporting students who face difficulties with *the problem worksheets* by making specific suggestions (e.g. guiding questions);
>
> (3) *poll worksheets*: worksheets prompting a poll among proposed options".

The authors identified feedback strategies (Table 5) the teacher may adopt to give feedback to students (Cusi, Morselli& Sabena, 2018, p. 3466). These strategies are employed in the class discussion that is organized by the teacher after the group work on worksheets.

Table 5:

Revoicing	When the teacher mirrors one student's intervention so as to draw the attention on it. Often, during the revoicing, the teacher stresses with voice intonation some crucial words of the sentence she is mirroring. Rephrasing takes place when the teacher reformulates the intervention of one student, with the double aim of drawing the attention of the class and making the intervention more intelligible to everybody.
Rephrasing	Rephrasing takes place when the teacher reformulates the intervention of one student, with the double aim of drawing the attention of the class and making the intervention more intelligible to everybody. Rephrasing is applied when the teacher feels that the intervention could be useful but needs to be communicated in a better way so as to become a resource for the others. […] The revoicing and rephrasing strategies […] turn one student (the author of the intervention) into a resource for the class.
Rephrasing with scaffolding	When the teacher, besides rephrasing, adds some elements to guide the students' work.
Relaunching	When the teacher reacts to a student's intervention, which (s)he considers interesting for the class, not giving a direct feedback, but posing a connected question. In this way, by relaunching the teacher provides an implicit feedback […] on the student's intervention,

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	suggesting that the issue is interesting and worth to be deepened or, conversely, has some problematic points and should be reworked on.
Contrasting	Contrasting takes place when the teacher draws the attention on two or more interventions, representing two different positions, so as to promote a comparison. By contrasting, […] the authors of the two positions may be resource for the class as well as responsible of their own learning.

We draw from the FaSMEd experience the idea of creating classroom activities in the formative assessment perspective, which may promote inclusion.

3. Design

In the subsections 3.1-3.5, the activities of the intervention tool are presented in detail**.**

3.1 Difficulties identified through the B2 questionnaire

We detect difficulties in the following item of B2:

Calculate the values of thefollowingexpressions
$2^7 \times 2^3 =$…..

Difficulties are related to remembering and applying the formula of multiplication of values with the same base and the definition of exponent.

3.2 Cognitive area and math domain of interest

The difficulties detected are linked to the cognitive domain of Memory and in the domain of *Arithmetic.*

Memory (retrieval and processing):

	Arithmetic	Geometry	Algebra
Memory	Calculate the values of thefollowingexpressions $2^7 \times 2^3 =$…..		
Reasoning			
Visuo-spatial			

3.3 Educational Aims

The intervention tool is aimed at support memory in multiplying powers, which have the same base.

3.4 Addressing to Student /class

The Intervention tool is composed of an activity that have to be carried out with class.

3.5 Educational activities: the Intervention Tool

In this subsection the activity is to be described in detail.

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Understanding the formula

The teacher asks the students to write the outcome of these examples as a power:

- 3× 3 × 3=.....
- a×a× a=.....

Then they should try to solve the following example where they don't know the number of elements a.

a × a × a × a × a × a × a × a × a .... × a = ...

If the answer is wrong, the teacher could ask students to discuss if they can guess a more efficient notation. This could introduce the need of more efficient notation and, then, the reason of power.

Then, the teacher asks the students to give the definition of power and show the base and exponent. This will clarify vocabulary and symbols.

If they do not remember it, the teacher gives the definition of power:

The expression $\underbrace{b \times b \times \cdots \times b}_{n}$ can be written as b^n. Where the b is called the **base** and the n is called the **exponent**.

The teacher also reminds basic rules:

- $x^1 = x$
- $x^0 = 1$
- $x^n * x^m = x^{n+m}$
- $x^n : x^m = x^{n-m}$
- $(x^n)^m = x^{n*m}$

And ask the students to color a base in black and an exponent in reads in an example:

- 5× 5× 5= 5^3

The teacher can use cards to improve students' memorizing of the symbolic components and syntax. The teacher encourages students to do similar exercise with different cards, which will be an alternative visual aid (UDL guidelines).

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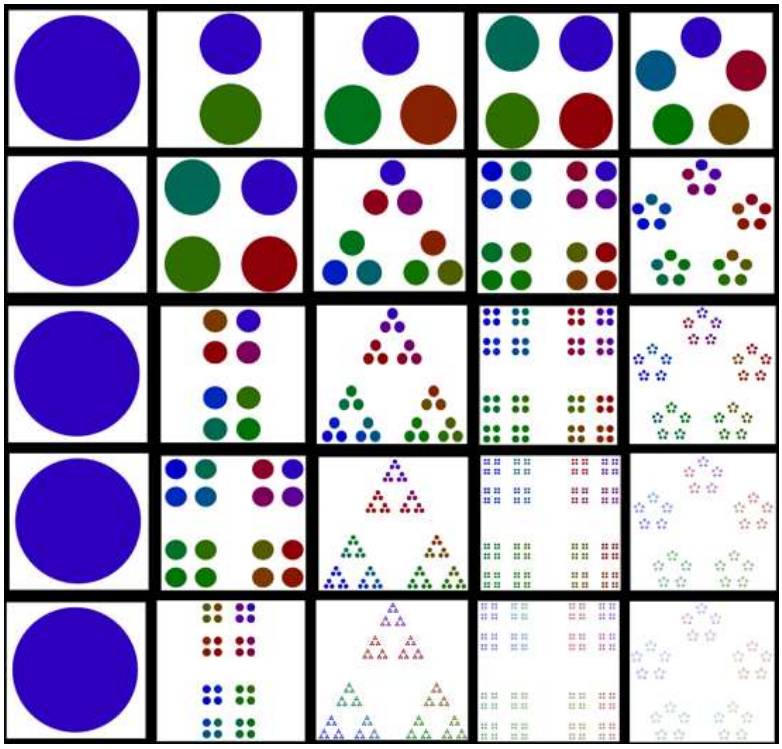
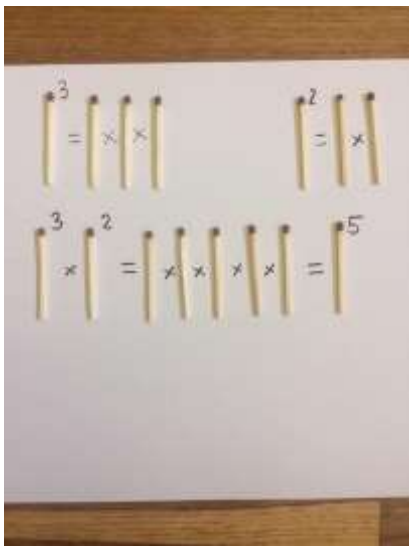
To help the students better visualise the idea of power, the teacher should show them the following picture and ask them to label each square with the power, e.g. 1^1, 2^1, 3^1, 4^1, 5^1 in the first row and 1^2, 2^2, 3^2, 4^2, 5^2 In the second and so on.



Then, teacher asks the students to apply the definition of power in an example:

- $2^7 \times 2^3 = (2\times 2\times 2\times 2\times 2\times 2\times 2)\times(2\times 2\times 2) = 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2 = 2^{10}$

Students compare their outcomes and discuss it, which will provide them feedback and allow stronger students to help weaker ones (FaSMEd). They can also do the exercise with matches.



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By doing this example, the students should be guided by a teacher so that they will be able tosee that when we multiply exponent expressions that have the same base, we get that base as many times as the sum of the exponents of the base. The activity will support decoding of mathematical notation (UDL guidelines).

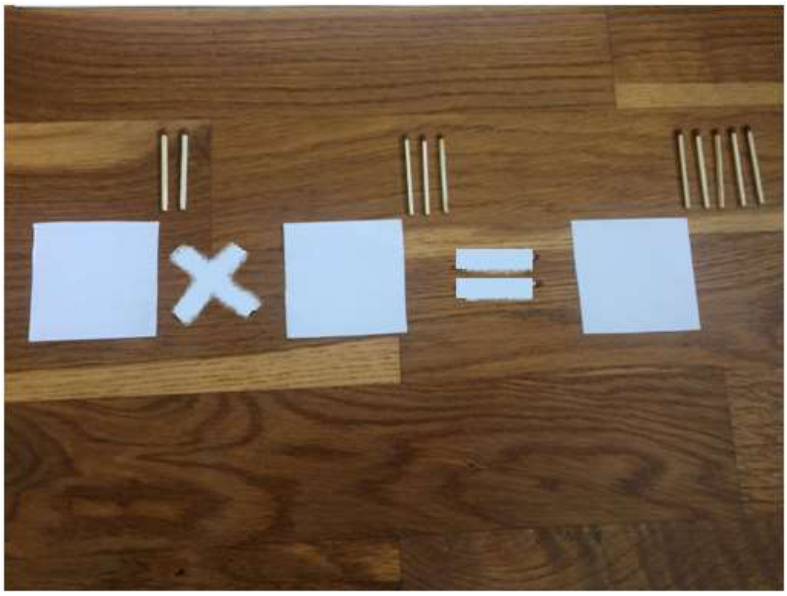
And teacher asks the students to do he same with general property:

- $b^m \cdot b^n = b^{(m+n)}$

with the examples:

- $10^{19} \cdot 10^{23} = 10^{19+23} = 10^{42}$
- $3^7 \cdot 3^8 = 3^{7+8} = 3^{15}$
- $\mathbf{2}^7 \times \mathbf{2}^3 = \mathbf{2}^{7+3} = 2^{10}$
- $5^7 \times 5^2 =$
- $13^7 \cdot 13^8 =$

Using different colours for the bases and exponents students can do more practice using matches which will be a useful way of customising the display of information (UDL guidelines).



4. References

[1] Karagiannakis, G. N., Baccaglini-Frank, A. E., &Roussos, P. (2016). Detectingstrengths and weaknesses in learningmathematicsthrough a model classifyingmathematicalskills. AustralianJ. of Learning Difficulties, 21(2), 115–141. https://doi.org/10.1080/19404158.2017.1289963
[2] Workshop with Dr. GiannisKaragianakis in International
[3] European Project FasMed (https://research.ncl.ac.uk/fasmed/).
[4] Universal design for learning (UDL) principles (http://udlguidelines.cast.org/)

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