Project Number: 2018-1-IT02-KA201-048274

INTERVENTION TOOL

Supporting Memory in Geometrical Problems 1

1. Introduction

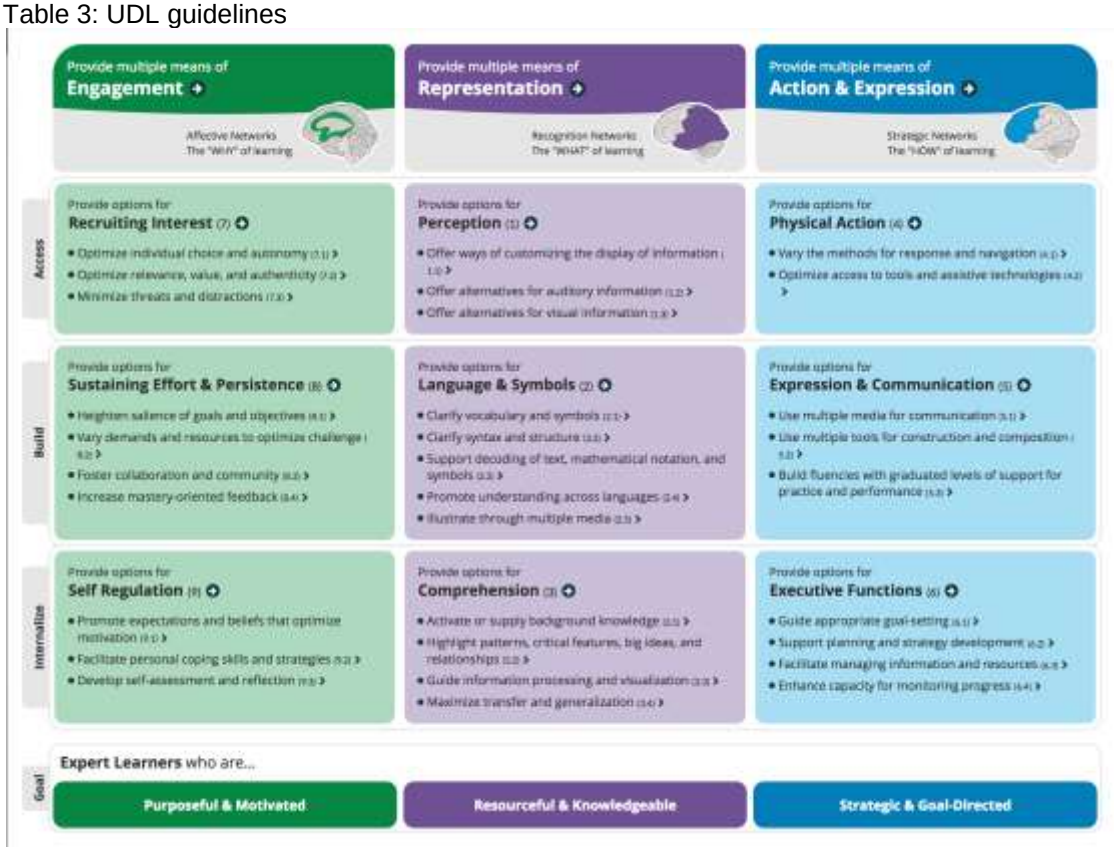
In order to develop educational activities aimed to support memory in geometry, we refer to some theoretical frameworks that will be described in the session 2. In session 3 the design of the educational activities is described. In particular, if the activities are addressed to students or the class, the educational aim of the activities, the Cognitive area and math domain of interest and the Mathematical objects in the areas of difficulties identified through the B2 questionnaire

2. Theoretical framework of reference

The theoretical references that helped us to design the following activities are:

1) **Universal design for learning (UDL) principles** (Table 3), a framework specifically conceived to design *inclusive* educational activities (http://udlguidelines.cast.org/)

Table 3: UDL guidelines



The Center for Applied Special Technology (CAST) has developed a comprehensive framework around the concept of Universal Design for Learning (UDL), with the aim of focusing research, development, and educational practice on understanding diversity and facilitating learning (Edyburn,

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2005). UDL includes a set of Principles, articulated in *Guidelines and Checkpoints*[1]. The research grounding UDL's framework is that "learners are highly variable in their response to instruction. [...]"
Thus, UDL focus on these individual differences as an important element to understanding and designing effective instruction for learning.
To this aim, UDL advances three foundational Principles: 1) provide multiple means of representation, 2) provide multiple means of action and expression, 3) provide multiple means of engagement. In particular, guidelines within the first principle have to do with means of perception involved in receiving certain information, and of "comprehension" of the information received. Instead, the guidelines within the second principle take into account the elaboration of information/ideas and their expression. Finally, the guidelines within the third principle deal with the domain of "affect" and "motivation", also essential in any educational activity.
For our analyses we will focus in particular on specific guidelines within the three Principles[2].
Guidelines within Principle 1 (provide multiple means of representation), suggest proposing different options for perception and offering support for decoding mathematical notation and symbols. Moreover, guidelines suggest the importance of providing options for comprehension highlighting patterns, critical features, big ideas, and relationships among mathematical notions. Finally, our analyses will give examples of how software AlNuSet can guide information processing, visualization, and manipulation, in order to maximize transfer and generalization.
Moreover, the guidelines from Principle 2 (provide multiple means of action and expression) suggest to offer different options for expression and communication supporting planning and strategy development. Finally, the guidelines from Principle 3 show how certain activities can recruit students' interest, optimizing individual choice and autonomy, and minimizing threats and distractions.
In the section 4 we will analyse examples of activities, classifying them both by the type of mathematical learning they are designed and the cognitive area they support. We will show how these examples have been designed on the UDL principles in order to make them inclusive and effective to overcame math difficulties identified through B2 questionnaire.

2) The European Project **FasMed,** that focused on formative assessment in mathematics and science, (https://research.ncl.ac.uk/fasmed/).
Formative assessment (FA) is conceived as a method of teaching where "evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited" (Black & Wiliam, 2009, p. 7).
FaSMEd project refers to Wiliam and Thompson (2007)'s study, that identifies five key strategies for FA practices in school setting: (a) *clarifying and sharing learning intentions and criteria for success;* (b) *engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding;* (c) *providing feedback that moves learners forward;* (d) *activating students as instructional resources for one another;* (e) *activating students as the owners of their own learning.*
The teacher, student's peers and the student him- or herself are the agents that activate these FA strategies.

[1] For a complete list of the principles, guidelines and checkpoints and a more extensive description of CAST's activities, visit http://www.udlcenter.org
[2] The items are taken from the interactive list at http://www.udlcenter.org/research/researchevidence

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Table 4: Formative assessment strategies

<table>
<tr><th></th><th>Where the learner is going</th><th>Where the learner is right now</th><th>How to get there</th></tr>
<tr><td>Teacher</td><td rowspan="3">1 Clarifying learning intentions and criteria for success

Understanding and sharing learning intentions and criteria for success

Understanding learning intentions and criteria for success</td><td>2 Engineering effective class-room discussions and other learning tasks that elicit evidence of student understanding</td><td>3 Providing feedback that moves learners forward</td></tr>
<tr><td>Peer</td><td colspan="2">4 Activating students as instructional resources for one another</td></tr>
<tr><td>Learner</td><td colspan="2">5 Activating students as the owners of their own learning</td></tr>
</table>

FaSMEd activities are organized in sequences, that encompass group work on worksheets and class discussion where selected group works are discussed by the whole class, under the orchestration of the teacher. Taking into account formative assessment strategies and technology functionalities, Cusi, Morselli & Sabena (2017, p. 758) designed three types of worksheets for the classroom activity:

> "(1) *problem worksheets:* worksheets introducing a problem and asking one or more questions involving the interpretation or the construction of the representation (verbal, symbolic, graphic, tabular) of the mathematical relation between two variables (e.g. interpreting a time-distance graph);
>
> (2) *helping worksheets*, aimed at supporting students who face difficulties with *the problem worksheets* by making specific suggestions (e.g. guiding questions);
>
> (3) *poll worksheets*: worksheets prompting a poll among proposed options".

The authors identified feedback strategies (Table 5) the teacher may adopt to give feedback to students (Cusi, Morselli & Sabena, 2018, p. 3466). These strategies are employed in the class discussion that is organized by the teacher after the group work on worksheets.

Table 5:

Revoicing	When the teacher mirrors one student's intervention so as to draw the attention on it. Often, during the revoicing, the teacher stresses with voice intonation some crucial words of the sentence she is mirroring. Rephrasing takes place when the teacher reformulates the intervention of one student, with the double aim of drawing the attention of the class and making the intervention more intelligible to everybody.
Rephrasing	Rephrasing takes place when the teacher reformulates the intervention of one student, with the double aim of drawing the attention of the class and making the intervention more intelligible to everybody. Rephrasing is applied when the teacher feels that the intervention could be useful but needs to be communicated in a better way so as to become a resource for the others. […] The revoicing and rephrasing strategies […] turn one student (the author of the intervention) into a resource for the class.
Rephrasing with scaffolding	When the teacher, besides rephrasing, adds some elements to guide the students' work.
Relaunching	When the teacher reacts to a student's intervention, which (s)he considers interesting for the class, not giving a direct feedback, but posing a connected question. In this way, by relaunching the teacher provides an implicit feedback […] on the student's intervention, suggesting that the issue is interesting and worth to be deepened or,

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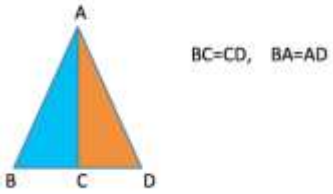
	conversely, has some problematic points and should be reworked on.
Contrasting	Contrasting takes place when the teacher draws the attention on two or more interventions, representing two different positions, so as to promote a comparison. By contrasting, […] the authors of the two positions may be resource for the class as well as responsible of their own learning.

We draw from the FaSMEd experience the idea of creating classroom activities in the formative assessment perspective, which may promote inclusion.

3. Design

3.1 Difficulties identified through the B2 questionnaire

We detect difficulties in the following item of B2:



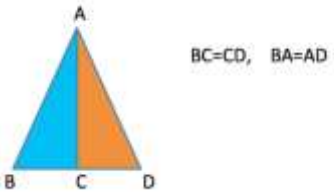
Which kind of triangle is CDA?
Which kind of triangle is BDA?

Difficulties are related to:

- Visual tracing of the text being read and of the attached drawing
- Recognition of the individual elements making up the figure
- Difficulty in memorizing information
- Difficulties in recalling, reproducing remembered information

3.2 Cognitive area and math domain of interest

The specific difficulties identified through B2 questionnaire is related to the domain of *Geometry*. *Memory* is the cognitive area involved.

	Arithmetic	**Geometry**	**Algebra**
Memory		 Which kind of triangle is CDA? Which kind of triangle is BDA?	

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Reasoning			
Visuo-spatial			

In the Table 2 the location of difficulties with respect to cognitive domain and mathematical area.
Table 2: The difficulties detected are linked to the cognitive area of *Memory* and in the domain of *Geometry*.

3.3 Educational Aims

The intervention tool is aimed at *Constructing strategies to retrieve geometric facts, to memorize them and to use them for reasoning.*

3.4 Addressing to Student /class

The intervention tool may be addressed to the single student.

3.5 Educational activities: the Intervention Tool

The teacher gives the student card with a problem described in 3.1 and asks the student:

- Read the problem with the triangle, please.
- How many triangles do you see in the drawing?
- Point out these triangles calling their vertices, please.
- BC=CD, BA=AD, so mark the same lengths in the same way.
  The student can mark them for example:

BC=CD, BA=AD

or

BC=CD, BA=AD

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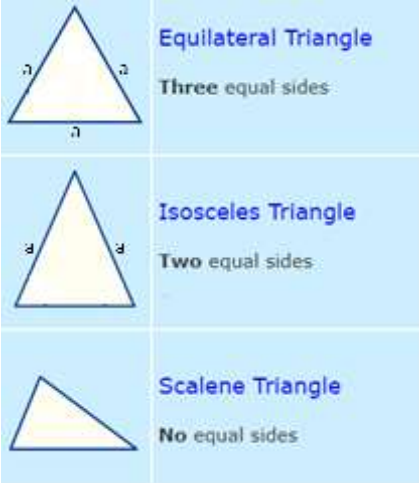
Then the teacher gives the student a card with a table



And says: There are three kinds of triangles if we are thinking about sides.

The teacher says:

- *Look at the triangle BDA.*
- *Does it have three or two equal sides? If you don't know, you can measure with a ruler and write the lengths next to the sides of the triangle.*

The student can measure with a ruler, and answer the questions.

The teacher says:

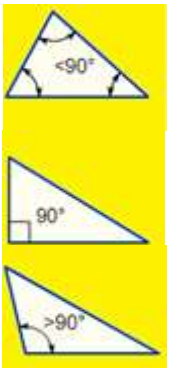
- *Now look to the card with three kinds of triangles if we are thinking about sides.*
- *What kind of triangle is triangle BDA? Write your answer and explain why you think so*.

If the students don't know the answer or can't explain their answer, the teachers should use the techniques of rephrasing or relaunching, e.g. Does the triangle have two or three equal sides? Is it a scalene triangle? This will promote a discussion which will elicit the understanding (FaSMEd).

Then the teacher says:

-*There are three kinds of triangles if we are thinking about angles*:

and gives the student a card with a table



Acute Triangle
All angles are less than 90°

Right Triangle
All angles are less than 90°

Obtuse Triangle
All angles are less than 90°

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Project Number: 2018-1-IT02-KA201-048274

The teacher says:
- *Look at the triangle CDA.*
- *Does it have three angles less than 90°? You can use protractor to check.*
- *Does it have right angle or angle more than 90°? You can use protractor to check in order to provide feedback.*

So, what kind of triangle is CDA if you are thinking about angles? You can use yellow table.

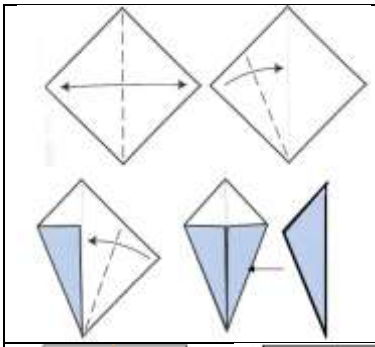
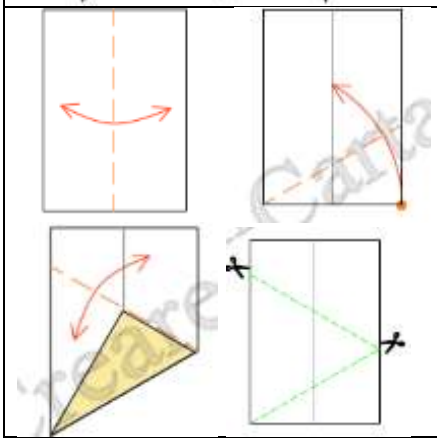
The student can also notice, that if BC=CD, BA=AD, then CA is median and altitude, so angle DCA is a right angle and CDA is Right Triangle.

Next the teacher says:
- *Now look at the triangle BDA.*
- *Using a ruler or a protractor, you can measure its sides and angles.*
- *The information in the tables will help you determine, what kind of triangle BDA is.*

The student is working to determine, what kind of triangle BDA is.

To support visualization in definition of triangles by angles or sides, the teacher can propose the dynamic approach by GeoGebra:
https://www.geogebra.org/m/xrdmybdw (in section: Rotation the triangle)
https://www.geogebra.org/m/aZaSAP3N#material/yrvXVSAw
https://www.geogebra.org/m/jFrz2Dsj

Through a kinesthetic approach, teacher propones an “origami” activity to visualize the ABD isoscel triangle as “composed” by two right triangles BCA and DCA.

	Starting with a square sheet Teacher asks students: - To fold the sheet as shown in the figure - to cut blue right triangles - to label each of their vertexes as in the model (BCA and CDA)
	Starting with a rectangular sheet Teacher asks students: - To fold the sheet as shown in the figure - to cut yellow right triangles - to label each of their vertexes as in the model (BCA and CDA)

After then, the teacher says:
-To be good in kinds of triangles you can try to memorize information (or to build tools that allow you to remember) about triangles placed in tables.

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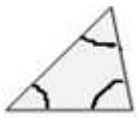
- For example you can do six cartons, as you see here:

One side

Another side

Acute Triangle 	All angles are less than 90°

-One carton to one kind of triangle, look:
(the teacher presents photos)







- Then you can look at one side of carton, and try to tell information which is on the another side. If you don't remember, you can reverse this carton and read information. If you will practice with your friend, you will be better in kinds of tringles. This will activate students as instructional resources for one another and help them own their learning (FaSMEd).

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4. Discussion through UDL guidelines about the above-mentioned activities

In *red* our comments to illustrate the connection between the principles of UDL and our activities

	Engagement	**Representation**	**Action & Expression**
	Affective Networks The "WHY" of Learning	Recognition Networks The "WHAT" of Learning	Strategic Networks The "HOW" of Learning
Access	**Recruiting Interest (7)** • Optimize individual choice and autonomy (7.1) • Optimize relevance, value, and authenticity (7.2) • Minimize threats and distractions (7.3) *Optimize individual choice and autonomy* *Optimize relevance, value, and authenticity* *Minimize threats and distractions.*	**Perception (1)** • Offer ways of customizing the display of information (1.1) • Offer alternatives for auditory information (1.2) • Offer alternatives for visual information (1.3) *Informations not only verbal, but visual and short-symolic* *Using different colours at picture and tables.*	**Physical Action (4)** • Vary the methods for response and navigation (4.1) • Optimize access to tools and assistive technologies (4.2) *Blue and yellow cartons with informations as simple to use sources of knowledge, the student can chose only one table to use at the moment or two tables-if he needs.*
Built	**Sustaining Effort & Persistence (8)** • Heighten salience of goals and objectives (8.1) • Vary demands and resources to optimize challenge (8.2) • Foster collaboration and community (8.3) • Increase mastery-oriented feedback (8.4)	**Language & Symbols (2)** • Clarify vocabulary and symbols (2.1) • Clarify syntax and structure (2.2) • Support decoding of text, mathematical notation, and symbols (2.3) • Promote understanding across languages (2.4) • Illustrate through multiple media (2.5) *The information is presented by text, symbols and an alternative method-graphically, at the drawing.*	**Expression & Communication (5)** • Use multiple media for communication (5.1) • Use multiple tools for construction and composition (5.2) • Build fluencies with graduated levels of support for practice and performance (5.3) *The student gets experiences, that initially difficult task can be solved with the necessary information (blue & yellow tables) so it is worthwhile to improve the methods of remembering information* *The student can create cartons need to effective systematic learning of triangle kinds and try to use them.*

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Internalize	**Self Regulation (9)** • Promote expectations and beliefs that optimize motivation (9.1) • Facilitate personal coping skills and strategies (9.2) • Develop self-assessment and reflection (9.3) *The student is confident that he or she will be able to solve the problem if he or she uses appriopriate information and methods.*	**Comprehension (3)** • Activate or supply background knowledge (3.1) • Highlight patterns, critical features, big ideas, and relationships (3.2) • Guide information processing and visualization (3.3) • Maximize transfer and generalization (3.4) *The student understands th e verbal and symbolic entri es in the task and is able to present them in a convenie nt graphic manner.*	**Executive Functions (6)** • Guide appropriate goal-setting (6.1) • Support planning and strategy development (6.2) • Facilitate managing information and resources (6.3) • Enhance capacity for monitoring progress (6.4) *The student is able to search for necessary information in tables (blue and yellow), to use tools to check hypotheses.* *The student is able to use a prepared set of fiches to learn triangle types and monitor progress in learning*
Goal	**Purpeseful & Motivated**	**Resourceful & Knowledgeable**	**Strategic & Goal-Directed**

5. References

[1] Karagiannakis, G. N., Baccaglini-Frank, A. E., & Roussos, P. (2016). Detecting strengths and weaknesses in learning mathematics through a model classifying mathematical skills. Australian J. of Learning Difficulties, 21(2), 115–141. https://doi.org/10.1080/19404158.2017.1289963
[2] Workshop with Dr. Giannis Karagianakis in International
[3] European Project FasMed (https://research.ncl.ac.uk/fasmed/).
[4] Universal design for learning (UDL) principles (http://udlguidelines.cast.org/)
[5] https://www.mathsisfun.com/triangle.html

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