Project Number: 2018-1-IT02-KA201-048274

INTERVENTION TOOL

Memorizing geometrical facts

1. Introduction

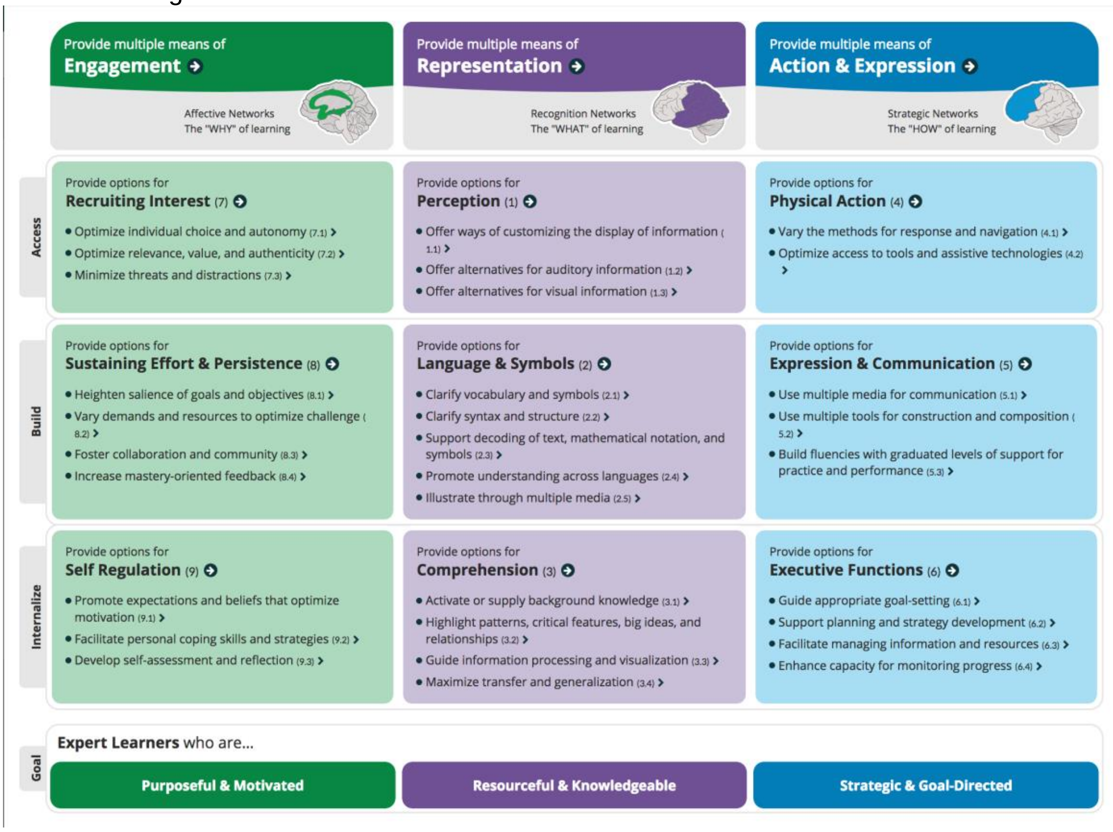
In this intervention tool we propose activities aimed at supporting the memorization of geometrical facts. The intervention is framed in the theoretical frameworks that are described in session 2, while in session 3 the design of the educational activities is described.

2. Theoretical framework of reference

The theoretical references that frame the design are:

1) **Universal design for learning (UDL) principles** (Table 3), a framework specifically conceived to design *inclusive* educational activities (http://udlguidelines.cast.org/)

Table 3: UDL guidelines



The Center for Applied Special Technology (CAST) has developed a comprehensive framework around the concept of Universal Design for Learning (UDL), with the aim of focusing research, development, and educational practice on understanding diversity and facilitating learning (Edyburn, 2005). UDL includes a set of Principles, articulated in *Guidelines and Checkpoints*[1]. The research grounding UDL's framework is that "learners are highly variable in their response to instruction. [...]"

[1] For a complete list of the principles, guidelines and checkpoints and a more extensive description of CAST's activities, visit http://www.udlcenter.org

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Thus, UDL focuses on these individual differences as an important element to understanding and designing effective instruction for learning.
To this aim, UDL advances three foundational Principles: 1) provide multiple means of representation, 2) provide multiple means of action and expression, 3) provide multiple means of engagement. In particular, guidelines within the first principle have to do with means of perception involved in receiving certain information, and of “comprehension” of the information received. Instead, the guidelines within the second principle take into account the elaboration of information/ideas and their expression. Finally, the guidelines within the third principle deal with the domain of “affect” and “motivation”, also essential in any educational activity.
For our analyses we will focus in particular on specific guidelines within the three Principles[2].
In order to characterize students' difficulties in geometry, we refer to the following elements of Karagiannakis' and colleagues' frame (Table 1), which dealt with Memory in retrieval of geometrical facts and geometrical processing: retrieval geometrical facts, remembering theorems, remembering hypothesis and thesis which are focusing on.

Table 1: Karagiannakis’s and colleagues’ frame: domains of the four-pronged model and sets of mathematical skills associated with each domain

Domain	Mathematical skills associated with the domain
Core number	Estimating accurately a small number of objects (up to 4); estimating approximately quantities; placing numbers on number lines; managing Arabic symbols; transcoding a number from one representation to another (analogical-Arabic-verbal); counting principles awareness
Memory (retrieval and processing)	Retrieving numerical facts; decoding terminology (numerator, denominator, isosceles, equilateral); remembering theorems and formulas; performing mental calculations fluently; remembering procedures and keeping track of steps
Reasoning	Grasping mathematical concepts, ideas and relations; understanding multiple steps in complex procedures/algorithms; grasping basic logical principles (conditionality – “if … then …” statements – commutativity, inversion); grasping the semantic structure of problems; (strategic) decision-making; generalizing
Visual-spatial	Interpreting and using spatial organization of representations of mathematical objects (for example, numbers in decimal positional notation, exponents, geometrical 2D and 3D figures or rotations); placing numbers on a number line; confusing Arabic numerals and mathematics symbols; performing written calculation when position is important (e.g. borrowing/carrying); interpreting graphs and tables

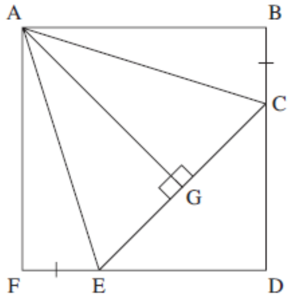
Visualization

Since this Intervention tool concerns geometrical activity, we also consider Duval’s theory on different cognitive apprehensions of figures, as the way to see, construct and describe a geometrical figure and its properties.
The Duval model is of particular interest as it is concerned with understanding the development of cognitive processes as revealed when solving geometry problems (Duval, 1998). Duval (1995) suggests an analytic theory for analysing thinking processes involved in a geometric activity.
As matter of fact, in Duval’s cognitive model of geometrical reasoning, the figure plays a key role:
- A figure gives us a figural representation of a geometrical situation which is shorter and easier to be understood than a representation with linguist speech.
- There are different cognitive apprehensions of figures through which Seeing, constructing and describing a geometrical figure and its properties:
  1. Perceptual apprehension
  2. Sequential apprehension
  3. Discursive apprehension

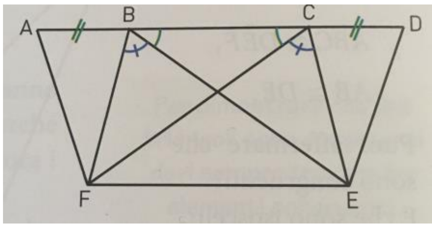
[2] The items are taken from the interactive list at http://www.udlcenter.org/research/researchevidence

4. Operative apprehension

1. Perceptual apprehension: It is about physical recognition (shape, representation, size, brightness, etc.) of a perceived figure. We should also discriminate and recognize sub-figures within the perceived figures since a relevant discrimination or recognition of these sub-figure units may help and give cues for problem solving in geometrical situations.



Or the following figure:



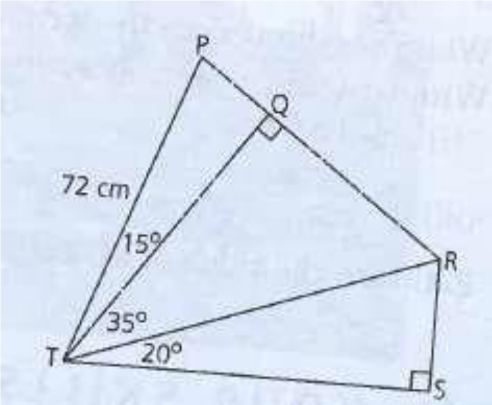
For example, the sub-figure FBE and FCE that are also superposed.

2. Sequential apprehension: It is about construction of a figure or description of its construction. Such construction depends on technical constraints and also mathematical properties since construction of a figure may merge different figural units. It is believed that construction can help recognition of relationships between mathematical properties and technical constraints.

3. Discursive apprehension: It is about (a) the ability to connect configuration(s) with geometric principles, (b) the ability to provide good description, explanation, argumentation, deduction, use of symbols, reasoning depending on statements made on perceptual apprehension, and (c) the ability to describe figures through geometric language/narrative texts

4. Operative apprehension: It is about making modification of a given figure in various ways to investigate others configurations:

- The metrological way: dividing the whole given figure into parts of various shapes and combine these parts in another figure or sub-figures;
- The optic way: varying the size of the figures; you can make a shape larger or narrower, or slant, the shapes can appear differently;
- The place way: varying the position or its orientation.



Formative assessment

Finally, we refer to the experience of the European Project FasMed, that focused on formative assessment in mathematics and science, (https://research.ncl.ac.uk/fasmed/).

Formative assessment (FA) is conceived as a method of teaching where "evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited" (Black & Wiliam, 2009, p. 7). FaSMEd project refers to Wiliam and Thompson (2007)'s study, that identifies five key strategies for FA practices in school setting: (a) clarifying and sharing learning intentions and criteria for success; (b) engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding; (c) providing feedback that moves learners forward; (d) activating students as instructional resources for one another; (e) activating students as the owners of their own learning. The teacher, student's peers and the student him- or herself are the agents that activate these FA strategies.

Formative assessment strategies

	Where the learner is going	Where the learner is right now	How to get there
Teacher	**1** Clarifying learning intentions and criteria for success	**2** Engineering effective class-room discussions and other learning tasks that elicit evidence of student understanding	**3** Providing feedback that moves learners forward
Peer	Understanding and sharing learning intentions and criteria for success	**4** Activating students as instructional resources for one another	
Learner	Understanding learning intentions and criteria for success	**5** Activating students as the owners of their own learning	

FaSMEd activities are organized in sequences, that encompass group work on worksheets and class discussion where selected group works are discussed by the whole class, under the orchestration of the teacher. Taking into account formative assessment strategies and technology functionalities, Cusi, Morselli & Sabena (2017, p. 758) designed three types of worksheets for the classroom activity:

> "(1) problem worksheets: worksheets introducing a problem and asking one or more questions involving the interpretation or the construction of the representation (verbal, symbolic, graphic, tabular) of the mathematical relation between two variables (e.g. interpreting a time-distance graph);

(2) helping worksheets, aimed at supporting students who face difficulties with the problem worksheets by making specific suggestions (e.g. guiding questions);

(3) poll worksheets: worksheets prompting a poll among proposed options”.

The authors identified feedback strategies (see Table below) the teacher may adopt to give feedback to students (Cusi, Morselli & Sabena, 2018, p. 3466). These strategies are employed in the class discussion that is organized by the teacher after the group work on worksheets.

Revoicing	When the teacher mirrors one student’s intervention so as to draw the attention on it. Often, during the revoicing, the teacher stresses with voice intonation some crucial words of the sentence she is mirroring. Rephrasing takes place when the teacher reformulates the intervention of one student, with the double aim of drawing the attention of the class and making the intervention more intelligible to everybody.
Rephrasing	Rephrasing takes place when the teacher reformulates the intervention of one student, with the double aim of drawing the attention of the class and making the intervention more intelligible to everybody. Rephrasing is applied when the teacher feels that the intervention could be useful but needs to be communicated in a better way so as to become a resource for the others. […] The revoicing and rephrasing strategies […] turn one student (the author of the intervention) into a resource for the class.
Rephrasing with scaffolding	When the teacher, besides rephrasing, adds some elements to guide the students’ work.
Relaunching	When the teacher reacts to a student’s intervention, which (s)he considers interesting for the class, not giving a direct feedback, but posing a connected question. In this way, by relaunching the teacher provides an implicit feedback […] on the student’s intervention, suggesting that the issue is interesting and worth to be deepened or, conversely, has some problematic points and should be reworked on.
Contrasting	Contrasting takes place when the teacher draws the attention on two or more interventions, representing two different positions, so as to promote a comparison. By contrasting, […] the authors of the two positions may be resource for the class as well as responsible of their own learning.

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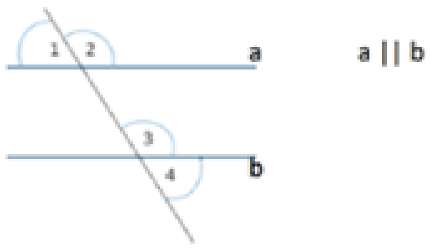
3. Design

3.1 Difficulties identified through the B2 questionnaire

The proposed intervention tool is aimed at supporting students who experiences difficulties in recovering geometrical facts. This is linked for instance to difficulties in the following item of questionnaire B2:

2. The sum of the interior angles of a triangle is equal to…..

3.



Which sentences are true?

a. Angles 1 and 4 are equal
b. Angles 2 and 3 have the sum 180°
c. Angles 1 and 2 have the sum 180°
d. Angle 3 is greater that angle 2

More specifically, difficulties are related to the recovery of geometrical facts and to the interpretation of the figure (visualizing angles and interpreting their numerical code)

3.2 Cognitive area and math domain of interest

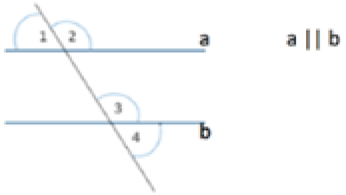
The area of difficulties identified through the B2 questionnaire is related to the domain of *Geometry*.

Memory is the cognitive area involved.

In Table 1 the location of difficulties with respect to cognitive domain and mathematical area.

Table 1: The difficulties detected are linked to the cognitive domain of *Memory* and in the domain of *Geometry*

	Arithmetic	**Geometry**	**Algebra**
Memory		2. The sum of the interior angles of a triangle is equal to…..	

		 Which sentences are true? a. Angles 1 and 4 are equal b. Angles 2 and 3 have the sum 180° c. Angles 1 and 2 have the sum 180° d. Angle 3 is greater that angle 2	
Reasoning			
Visuo-spatial			

3.3 Educational Aims

The intervention tool is aimed at *Constructing strategies in order to retrieve geometric facts and keep them in memory in order to use them for reasoning*

3.4 Addressing to Student /class

The activity that is suggested by the Intervention tool can to be proposed to the single student or to the whole class.

3.5 Educational activities: the Intervention Tool

The intervention tool is aimed at supporting memorization and retrieving of geometric facts concerning parallel lines and related angles. To this aim, we propose to use a dynamic geometry software (Geogebra) that allows to explore dynamically a geometric figure and detecting regularities.

Task 1

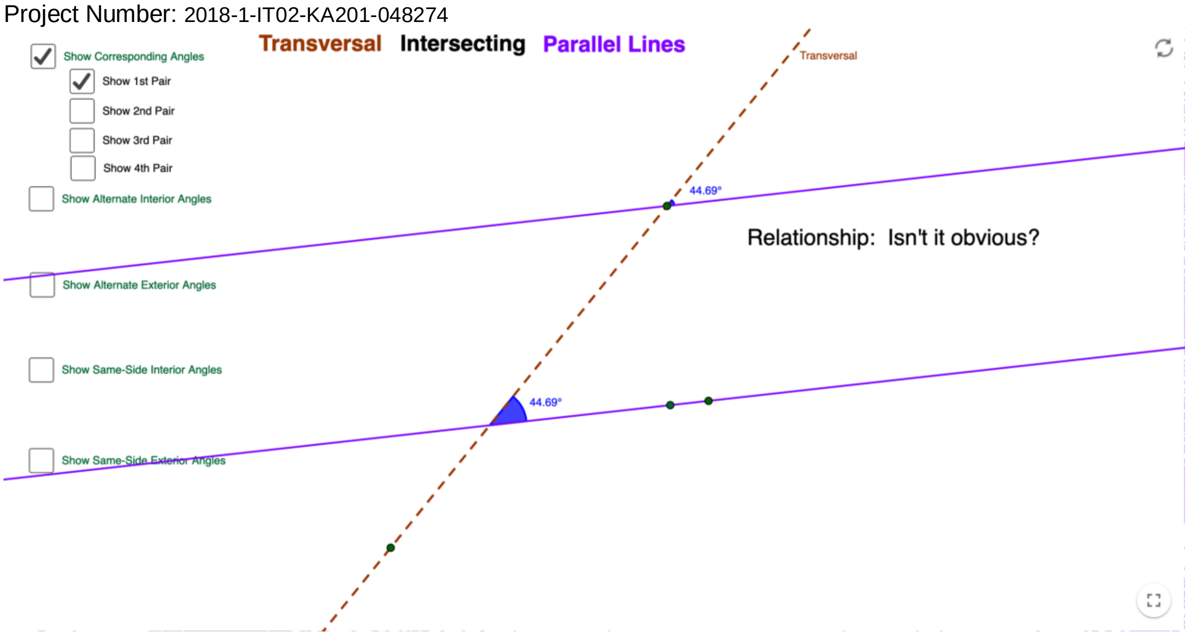
We refer to the activity that is proposed on the Geogebra site: https://www.geogebra.org/m/nfb2rtys

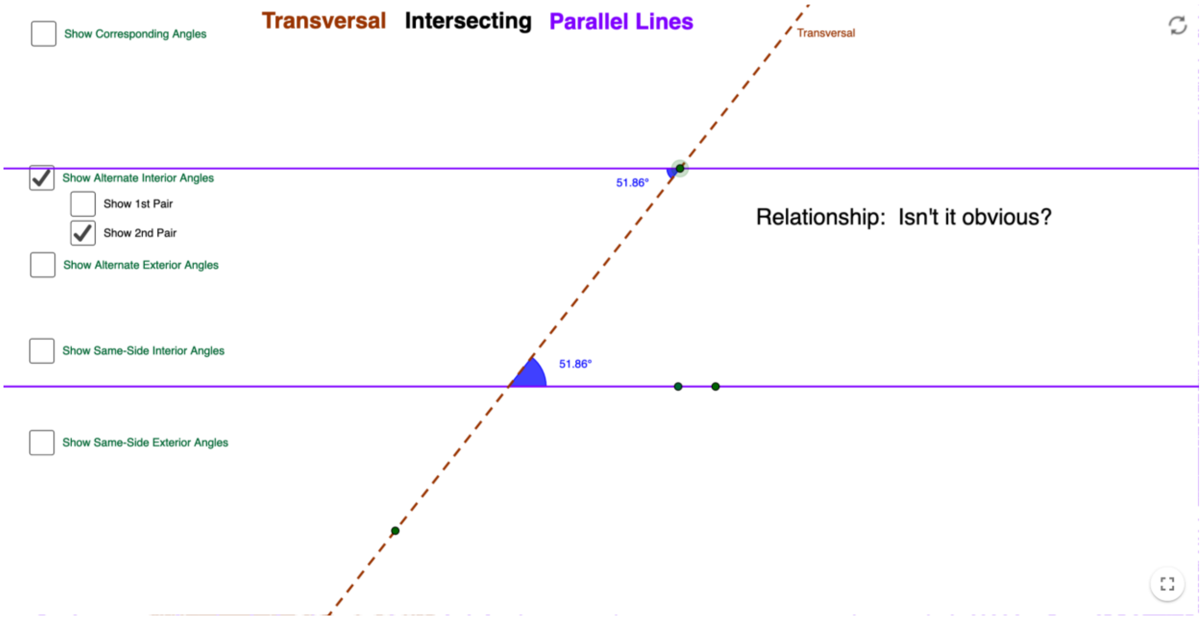
Students are asked to explore the dynamic figure and answer the questions that are proposed. The idea is that, by dragging the geometric elements, relevant properties are outlined. Two screenshots from the activity show that students can explore the figure dynamically (dragging the green points, parallel lines and/or the transversal line move) and measure many couples of angles, so as to detect relevant relationships.

Task 2

Once identified relevant relationships, students are asked to screenshot figure and paste them on a virtual blackboard (such as padlet, https://padlet.com/dashboard). For each figure, students are asked to write down a description of the observed relationship.

Once all the students have written down their descriptions, the teacher may promote a class discussion to fix the relevant properties.





Discussion through UDL guidelines about the above-mentioned activities

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The proposed activity is in line with the principles of UDL. We insert our comments in red within the UDL table.

Table 3: Analysis of the activities through the Table of UDL principles.

Engagement	*Representation*	*Action & Expression*
Recruiting interest *Optimize individual choice and autonomy* *Optimize relevance, value, and authenticity* *Minimize threats and distractions*	***Perception*** Offer ways of customizing the display of information Offer alternatives for auditory information Offer alternatives for visual information Information is displayed by means of different registers (visual non verbal, verbal and symbolic)	***Physical Action*** Vary the methods for response and navigation Optimize access to tools and assistive technologies *Geogebra supports the physical action on the figural objects*
Sustaining effort Persistence Heighten salience of goals and objectives Vary demands and resources to optimize challenge Foster collaboration and community Increase mastery-oriented feedback Vary demands and resources to optimize challenge Foster collaboration and community *Feedbacks of software support engagement and motivation with respect to the elaboration of the solution of the task*	***Language & Symbols*** Clarify vocabulary and symbols Clarify syntax and structure Offer alternative language and symbols to decode information and to work on the information *This is promoted by the use of different registers of representation: figural non verbal on the drawing, colors, written description* Support decoding of text, mathematical notation, and symbols Promote understanding across languages Illustrate through multiple media *This is promoted by the use of dynamic geometry software such as GeoGebra. In the final part of the activity, students are also asked to describe in natural language what they discovered.* Support decoding of text,	***Expression Communication*** Use multiple media for communication Use multiple tools for construction and composition Build fluencies with graduated levels of support for practice and performance To use different registers in order to communicate *In the final part of the activity, students are asked to describe in natural language what they discovered.*

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	math notation and symbols *This is promoted by the visualization of hypotheses by drawing realized by GeoGebra*	
Self Regulation	***Comprehension***	***Executive functions***
Promote expectations and beliefs that optimize motivation *Facilitate personal coping skills and strategies* Develop self-assessment and reflection	Activate or supply background knowledge Highlight patterns, critical features, big ideas, and relationships (checkpoint 3.2) Guide information processing and visualization Maximize transfer and generalization *To support generalization, the tasks suggest to visualize drawings on GeoGebra. Indeed, the drag function of GeoGebra allows students to identify invariants of the figure and recover suitable theorem in order to develop the required proof.* Perception, language and symbols, comprehension (Constructing useable knowledge, knowledge that is accessible for future decision-making, depends not upon merely perceiving information, but upon active “information processing skills”)	*Guide appropriate goal-setting* Support planning and strategy development Facilitate managing information and resources *Students work on the dynamic geometry software, afterwards they are led to organize their discoveries* Enhance capacity for monitoring progress

Discussion through the FaSMEd model about the above-mentioned activities

The proposed activity also promotes Formative Assessment Strategies, as described in the FaSMed project.

More specifically, each student is led to be responsible of his/her own learning when writing down on the virtual blackboard his/her discoveries. The teacher organizes a class discussion, when he/she provides feedback to the students and each student may also act as resource for the mates.

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5. References

[1]Karagiannakis, G. N., Baccaglini-Frank, A. E., & Roussos, P. (2016). Detecting strengths and weaknesses in learning mathematics through a model classifying mathematical skills. Australian J. of Learning Difficulties, 21(2), 115–141.
[2]Duval, R.: 1995, 'Geometrical Pictures: Kinds of representation and specific processing', in
[3]R. Suttherland and J. Mason (eds.), Exploiting Mental Imagery with Computers in Mathematics Education, Springer, Berlin, pp. 142–157.
[4]Duval, R.: 1998, 'Geometry from a cognitive point a view', in C. Mammana and V. Villani (eds.), Perspectives on the Teaching of Geometry for the 21st Century, Kluwer Academic Publishers, Dordrecht, pp. 37–52.
[5]UDL Principles: http://udlguidelines.cast.org/
[6]GeoGebra materials:
https://www.geogebra.org/m/rSuyACJC;
https://www.geogebra.org/m/rSuyACJC#material/R6by3BuA