Project Number: 2018-1IT02KA201048274

INTERVENTION TOOL

Arithmetical Reasoning

1. Introduction

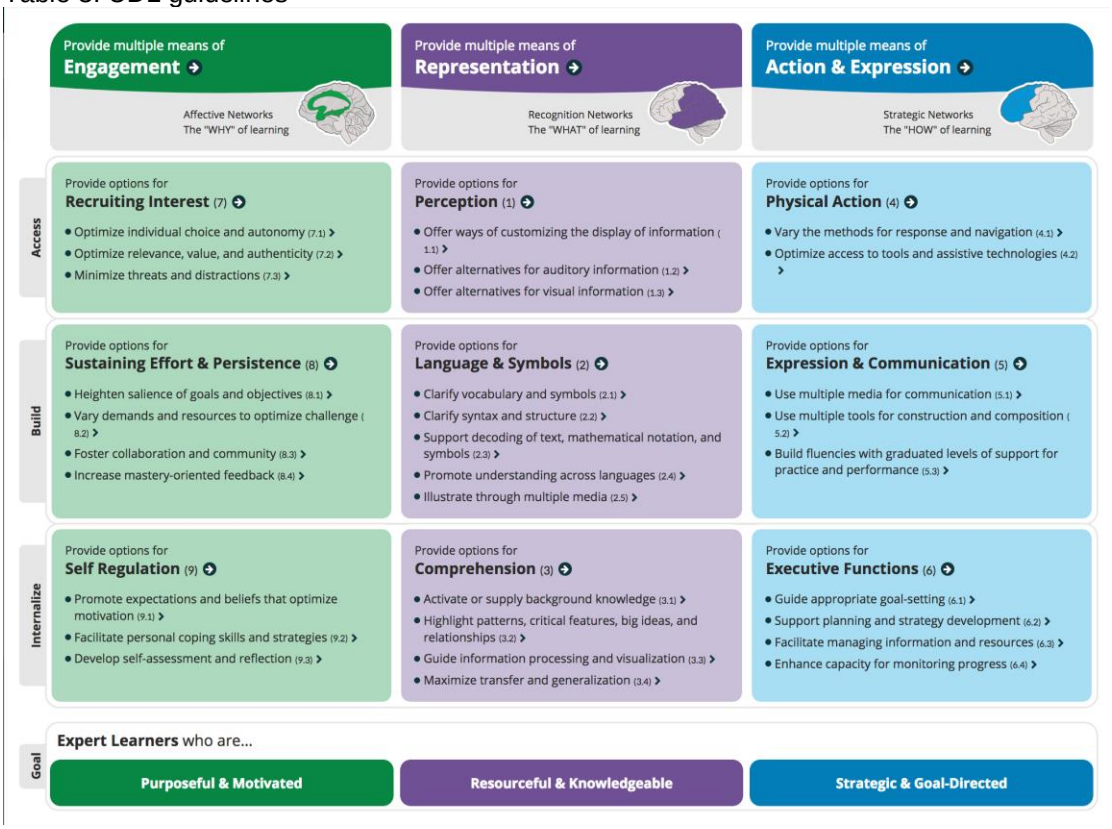
In order to develop a set of educational activities aimed to develop reasoning in the arithmetic domain, we refer to some theoretical frameworks that will be described in the session 2.
In session 3 the design of the educational activities is described. In particular, if the activities are addressed to a student or the class, the educational aim of the activities, the Cognitive area and math domain of interest and the Mathematical objects in the areas of difficulties identified through the B2 questionnaire

2. Theoretical framework of reference

The theoretical references that helped us to design the following activities are:

1) **Universal design for learning (UDL) principles** (Table 3), a framework specifically conceived to design *inclusive* educational activities (http://udlguidelines.cast.org/)

Table 3: UDL guidelines



The Center for Applied Special Technology (CAST) has developed a comprehensive framework around the concept of Universal Design for Learning (UDL), with the aim of focusing research, development, and educational practice on understanding diversity and facilitating learning

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(Edyburn, 2005). UDL includes a set of Principles, articulated in *Guidelines and Checkpoints*[1]. The research grounding UDL's framework is that "learners are highly variable in their response to instruction. [...]"
Thus, UDL focus on these individual differences as an important element to understanding and designing effective instruction for learning.
To this aim, UDL advances three foundational Principles: 1) provide multiple means of representation, 2) provide multiple means of action and expression, 3) provide multiple means of engagement. In particular, guidelines within the first principle have to do with means of perception involved in receiving certain information, and of "comprehension" of the information received. Instead, the guidelines within the second principle take into account the elaboration of information/ideas and their expression. Finally, the guidelines within the third principle deal with the domain of "affect" and "motivation", also essential in any educational activity.
For our analyses we will focus in particular on specific guidelines within the three Principles[2].
Guidelines within Principle 1 (provide multiple means of representation), suggest proposing different options for perception and offering support for decoding mathematical notation and symbols. Moreover, guidelines suggest the importance of providing options for comprehension highlighting patterns, critical features, big ideas, and relationships among mathematical notions. Finally, our analyses will give examples of how software AlNuSet can guide information processing, visualization, and manipulation, in order to maximize transfer and generalization.
Moreover, the guidelines from Principle 2 (provide multiple means of action and expression) suggest to offer different options for expression and communication supporting planning and strategy development. Finally, the guidelines from Principle 3 show how certain activities can recruit students' interest, optimizing individual choice and autonomy, and minimizing threats and distractions.
In the section 4 we will analyse examples of activities, classifying them both by the type of mathematical learning they are designed and the cognitive area they support. We will show how these examples have been designed on the UDL principles in order to make them inclusive and effective to overcame math difficulties identified through B2 questionnaire.

3. Design

3.1 Difficulties identified through the B2 questionnaire

We detect difficulties in the following item of B2:

Sara received 24 euros as a gift, Marta received 6 euros less.
How many euros have the two girls in total?

These difficulties are related to reasoning in the arithmetic domain.

3.2 Cognitive area and math domain of interest

The area of difficulties identified through the B2 questionnaire is related to the domain of *Arithmetic*. In particular, the difficulties are related to the construction of the reasoning about a comparison of money. Thus, *Reasoning* is the cognitive area involved (Table 1).

[1] For a complete list of the principles, guidelines and checkpoints and a more extensive description of CAST's activities, visit http://www.udlcenter.org
[2] The items are taken from the interactive list at http://www.udlcenter.org/research/researchevidence

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Table 1: The difficulties detected are linked to the cognitive domain of *Reasoning* and in the domain of *Arithmetic*

	Arithmetic	Geometry	Algebra
Memory			
Reasoning	Sara received 24 euros as a gift, Marta received 6 euros less. How many euros have the two girls in total?		
Visuospatial			

3.3 Educational Aims

Develop reasoning in arithmetic about comparison of quantities (money).

3.4 Addressing to Student /class

The Intervention tool is articulated in a set of activities that have to be carried out with all the class, in a perspective of inclusion.

3.5 Educational activities: the Intervention Tool

The teaching sequences are conceived to address specific learning difficulty, within an inclusive perspective. They play the role of cognitive training where the student is led to use a special model (bar model) focused on the comparison of quantities. This educational tool is inspired to the bar model of Singapore approach to mathematics (see http://thesingaporemaths.com/index.html)

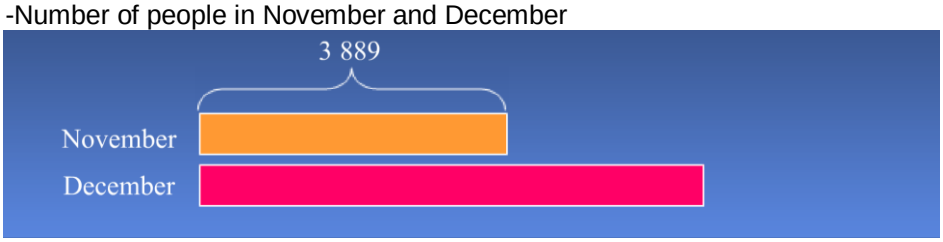
Activity 1

1. Question

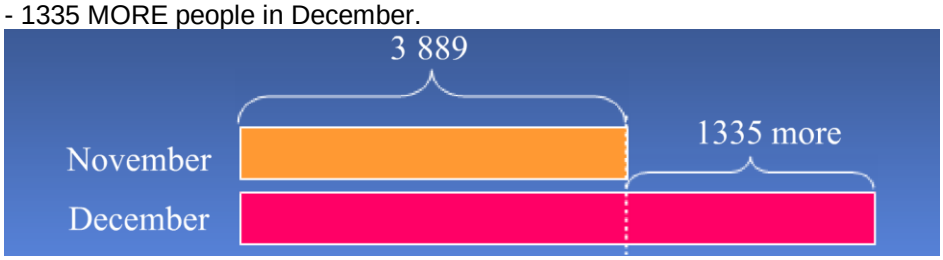
3889 people flew to San Francisco from Brazil in November. In December the number of people who flew to San Francisco from Brazil was 1335 more than in November. How many people went to San Francisco from Brazil in the two months?

2. The model (bar model) is drawn to present data:

-Number of people in November and December



- 1335 MORE people in December.



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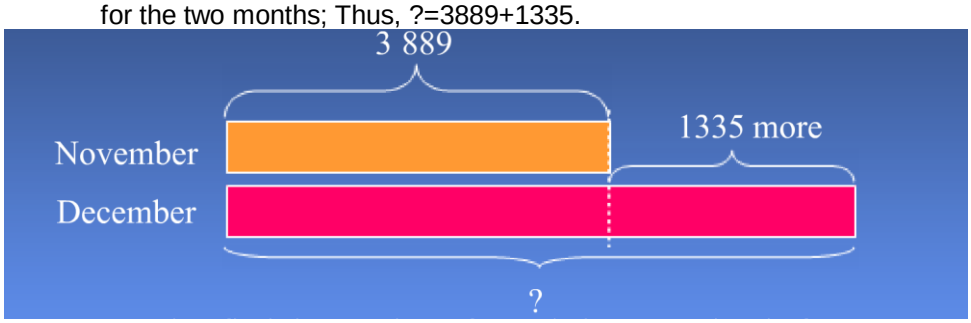
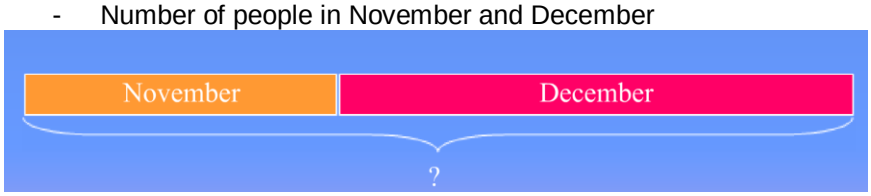
- You need to find the number of people in December before you can find the total of people for the two months; Thus, ?=3889+1335.



- Number of people in November and December



? = 3889 + 5224

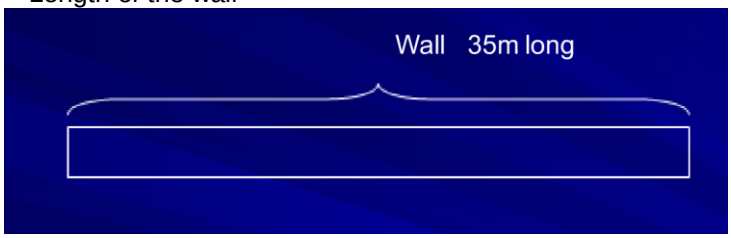
Activity 2

1. Question

A painter has to paint a 35 meters long wall. He painted 8.4 meters in the first hours and twice this length in the second hour. How many more meters of the wall must he paint to complete the job?

2. The model (bar model) is drawn to present data:

- Length of the wall



- How many meters the painter painted in the first hour

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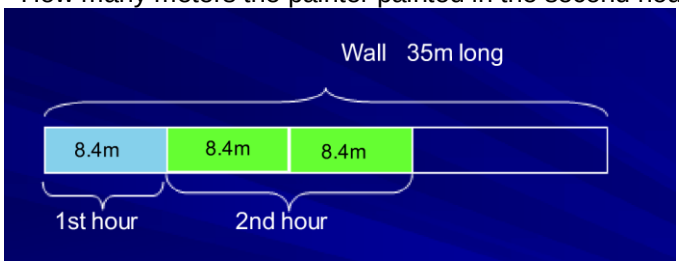
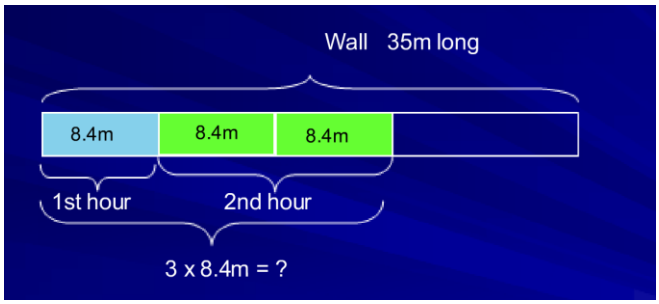
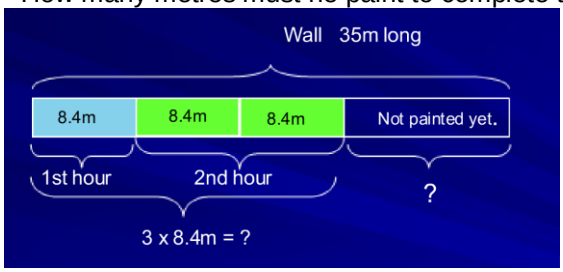
- How many meters the painter painted in the second hour



- How many meters the painter painted in three hours



- How many metres must he paint to complete the job?



? (Not painted yet) = 35m - (3x8,4m)

Collective discussion

3.6 Discussion through UDL guidelines about the above-mentioned activities

We observe that the same educational aim of constructing a reasoning in *Arithmetic* is approached in different ways by acting on the three principles of UDL (Table 7, in *red* our comments to illustrate the connection between the principles and our activities).

Table 7: Analysis of the activities through the Table of UDL principles.

Engagement	***Representation***	***Action & Expression***
Recruiting interest	***Perception***	***Physical Action***
Optimize individual choice and autonomy	Offer ways of customizing the display of information	Vary the methods for response and navigation
Optimize relevance, value, and authenticity	Offer alternatives for visual information	
Minimize threats and distractions	Different registers through which information are displayed (visual; symbolic)	

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Sustaining effort Persistence **Heighten salience of goals and objectives** **Increase mastery-oriented feedback** **Vary demands and resources to optimize challenge** **Foster collaboration and community** **Oriented feedbacks support engagement and motivation with respect the elaboration of the solution of the task**	***Language & Symbols*** Clarify vocabulary and symbols Clarify syntax and structure Offer alternative language and symbols to decode information and to work on the information *This is offered both by bar model and by symbolic non formal language (use of brackets, question point…)* Support decoding of text, mathematical notation, and symbols Promote understanding across languages	***Expression Communication*** Use multiple tools for construction and composition Build fluencies with graduated levels of support for practice and performance To use different registers in order to communicate *This is promoted by the use of terms and simbols that are alternative to the formal ones to speak about mathematical objects.* *For instance, brackets to put attention on many data, question point to put attention of unknown …*
Self Regulation **Promote expectations and beliefs that optimize motivation** **Facilitate personal coping skills and strategies** **Develop self-assessment and reflection** ***Formative assessment strategies, as discussed in section 2, may help self-assessment and reflection. More specifically, the teacher may provide different types of feedback***	***Comprehension*** Activate or supply background knowledge Highlight patterns, critical features, big ideas, and relationships (checkpoint 3.2) Guide information processing and visualization Maximize transfer and generalization Perception, language and symbols, comprehension (Constructing useable knowledge, knowledge that is accessible for future decision-making, depends not upon merely perceiving information, but upon active “information processing skills”)	***Executive functions*** *Guide appropriate goal-setting* *The use of bar model may also be a support for memory. Bar model guide students’ process of inquiry.* *Support planning and strategy development* *Facilitate managing information and resources* *Enhance capacity for monitoring progress*

This allows students to construct *meaning for the arithmetic* notions at stake.

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5. References

[1]http://thesingaporemaths.com/Index
[2]Karagiannakis, G. N., Baccaglini-Frank, A. E., & Roussos, P. (2016). Detecting strengths and weaknesses in learning mathematics through a model classifying mathematical skills. Australian J. of Learning Difficulties, 21(2), 115–141.
[3]UDL Principles: http://udlguidelines.cast.org/

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