Project number: 2018-1-IT02-KA201-048274

Intervention Tool

Proving Theorems – advanced level

1. Introduction

The intervention tool is conceived to address specific difficulties related to the mathematical domain of geometry and the cognitive domain of reasoning. By means of the intervention tool, that is conceived for all the class, the students may reflect on the proving process, with specific reference to crucial steps such as understanding the text, identifying hypothesis and thesis, representing hypotheses on the figure, organizing proof as a sequence of logically connected statements.

We suggest to consider the intervention tool "Proving – intermediate level" before starting this one. The tool has the same educational aim, with increasing difficulty concerning the statement to be proved.

The tool consists in a series of questions the teacher may pose to the students during a class discussion. Questions may be projected on the whiteboard. If the students have at disposal tablets or computers with internet connection, the questions can be administered by means of an interactive response system (e.g. Socrative, Mentimeter).

2. Theoretical framework of reference

We recall here Karagiannakis's and colleagues' frame (Table 1), which helps to characterize students' difficulties in mathematics.

Table 1: Karagiannakis's and colleagues' frame: domains of the four-pronged model and sets of mathematical skills associated with each domain

Domain	Mathematical skills associated with the domain
Core number	Estimating accurately a small number of objects (up to 4); estimating approximately quantities; placing numbers on number lines; managing Arabic symbols; transcoding a number from one representation to another (analogical-Arabic-verbal); counting principles awareness
Memory (retrieval and processing)	Retrieving numerical facts; decoding terminology (numerator, denominator, isosceles, equilateral); remembering theorems and formulas; performing mental calculations fluently; remembering procedures and keeping track of steps
Reasoning	Grasping mathematical concepts, ideas and relations; understanding multiple steps in complex procedures/algorithms; grasping basic logical principles (conditionality – "if … then …" statements – commutativity, inversion); grasping the semantic structure of problems; (strategic) decision-making; generalizing
Visual-spatial	Interpreting and using spatial organization of representations of mathematical objects (for example, numbers in decimal positional notation, exponents, geometrical 2D and 3D figures or rotations); placing numbers on a number line; confusing Arabic numerals and mathematics symbols; performing written calculation when position is important (e.g. borrowing/carrying); interpreting graphs and tables

We also recall that, when constructing B2, we chose questions that were related to the cognitive areas as well to three mathematical domains: arithmetic, geometry, algebra (Core number is not related to all cognitive areas). As a result, we proposed questions that were located in some cells of the following table (Table 2):

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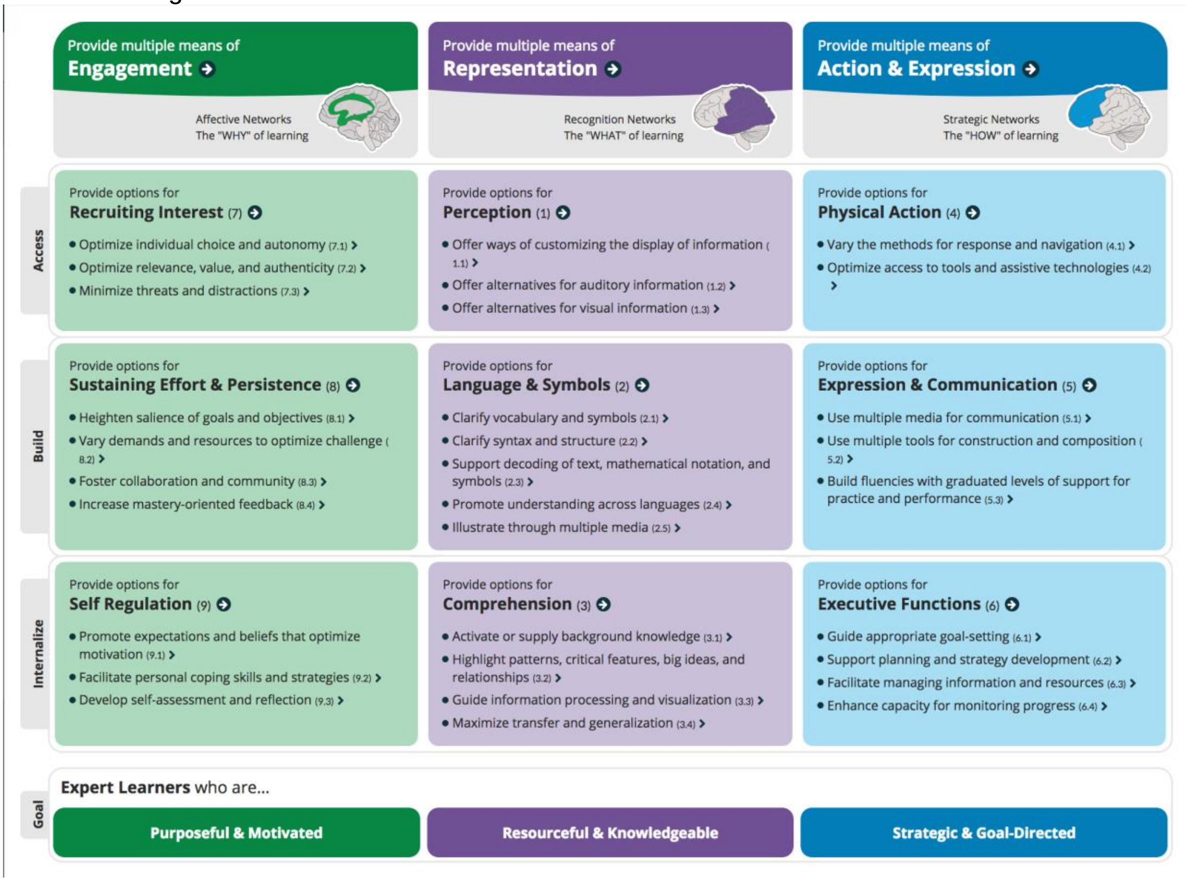
Table 2: Double relation between cognitive areas (memory, reasoning and visuo-spatial) and mathematical domains (arithmetic, geometry, algebra).

	Arithmetic	Geometry	Algebra
Memory			
Reasoning			
Visuo-spatial			

Here we present additional theoretical references that helped us to design the intervention tools.
First of all, we refer to the Universal design for learning (UDL) principles (Table 3), a framework specifically conceived to design inclusive educational activities (http://udlguidelines.cast.org/)

Table 3: UDL guidelines



The Center for Applied Special Technology (CAST) has developed a comprehensive framework around the concept of Universal Design for Learning (UDL), with the aim of focusing research, development, and educational practice on understanding diversity and facilitating learning (Edyburn, 2005). UDL includes a set of Principles, articulated in Guidelines and Checkpoints1. The research grounding UDL's framework is that "learners are highly variable in their response to instruction. [...]"
Thus, UDL focus on these individual differences as an important element to understanding and designing effective instruction for learning.
To this aim, UDL advances three foundational Principles: 1) provide multiple means of representation, 2) provide multiple means of action and expression 3) provide multiple means of engagement. In particular, guidelines within the first principle have to do with means of perception involved in

[1] For a complete list of the principles, guidelines and checkpoints and a more extensive description of CAST's activities, visit http://www.udlcenter.org

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receiving certain information, and of “comprehension” of the information received. Instead, the guidelines within the second principle take into account the elaboration of information/ideas and their expression. Finally, the guidelines within the third principle deal with the domain of “affect” and “motivation”, also essential in any educational activity.

Furthermore, we refer to the experience of the European Project FasMed, that focused on formative assessment in mathematics and science, (https://research.ncl.ac.uk/fasmed/).
Formative assessment (FA) is conceived as a method of teaching where “evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited” (Black & Wiliam, 2009, p. 7). FaSMEd project refers to Wiliam and Thompson (2007)’s study, that identifies five key strategies for FA practices in school setting: (a) clarifying and sharing learning intentions and criteria for success; (b) engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding; (c) providing feedback that moves learners forward; (d) activating students as instructional resources for one another; (e) activating students as the owners of their own learning. The teacher, student’s peers and the student him- or herself are the agents that activate these FA strategies.

Table 4: Formative assessment strategies

<table>
<tr><th></th><th>Where the learner is going</th><th>Where the learner is right now</th><th>How to get there</th></tr>
<tr><td>Teacher</td><td>1 Clarifying learning intentions and criteria for success</td><td>2 Engineering effective class-room discussions and other learning tasks that elicit evidence of student understanding</td><td>3 Providing feedback that moves learners forward</td></tr>
<tr><td>Peer</td><td>Understanding and sharing learning intentions and criteria for success</td><td colspan="2">4 Activating students as instructional resources for one another</td></tr>
<tr><td>Learner</td><td>Understanding learning intentions and criteria for success</td><td colspan="2">5 Activating students as the owners of their own learning</td></tr>
</table>

FaSMEd activities are organized in sequences, that encompass group work on worksheets and class discussion where selected group works are discussed by the whole class, under the orchestration of the teacher. Taking into account formative assessment strategies and technology functionalities, Cusi, Morselli & Sabena (2017, p. 758) designed three types of worksheets for the classroom activity:

> “(1) problem worksheets: worksheets introducing a problem and asking one or more questions involving the interpretation or the construction of the representation (verbal, symbolic, graphic, tabular) of the mathematical relation between two variables (e.g. interpreting a time-distance graph);
>
> (2) helping worksheets, aimed at supporting students who face difficulties with the problem worksheets by making specific suggestions (e.g. guiding questions);
>
> (3) poll worksheets: worksheets prompting a poll among proposed options”.

The authors identified feedback strategies (Table 5) the teacher may adopt to give feedback to students (Cusi, Morselli & Sabena, 2018, p. 3466). These strategies are employed in the class discussion that is organized by the teacher after the group work on worksheets.

Table 5:

Revoicing	When the teacher mirrors one student’s intervention so as to draw the attention on it. Often, during the revoicing, the teacher stresses with voice intonation some crucial words of the sentence she is mirroring. Rephrasing takes place when the teacher reformulates the intervention of

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	one student, with the double aim of drawing the attention of the class and making the intervention more intelligible to everybody.
Rephrasing	Rephrasing takes place when the teacher reformulates the intervention of one student, with the double aim of drawing the attention of the class and making the intervention more intelligible to everybody. Rephrasing is applied when the teacher feels that the intervention could be useful but needs to be communicated in a better way so as to become a resource for the others. […] The revoicing and rephrasing strategies […] turn one student (the author of the intervention) into a resource for the class.
Rephrasing with scaffolding	When the teacher, besides rephrasing, adds some elements to guide the students' work.
Relaunching	When the teacher reacts to a student's intervention, which (s)he considers interesting for the class, not giving a direct feedback, but posing a connected question. In this way, by relaunching the teacher provides an implicit feedback […] on the student's intervention, suggesting that the issue is interesting and worth to be deepened or, conversely, has some problematic points and should be reworked on.
Contrasting	Contrasting takes place when the teacher draws the attention on two or more interventions, representing two different positions, so as to promote a comparison. By contrasting, […] the authors of the two positions may be resource for the class as well as responsible of their own learning.

Moreover, we refer to research literature concerning the approach to proof in secondary school. Balacheff (1982) points out that the teaching of proofs and theorems should have the double aim of making students understand what a proof is, and learn to produce it. It is important that students understand the need of the proof, otherwise the risk is that they feel proof like a discourse aimed at showing to the teacher that the student possesses a given knowledge (proof risks to be seen as a part of the didactical contract, rather than as the means to validate the statement).

Balacheff distinguishes between pragmatic proofs and intellectual proofs. The first ones are based on the real action that is performed on the representations of the mathematical objects, while the second ones are based on the mental experiences and are carried out by means of language.

In particular, Balacheff illustrates:

- Naïf empirism (to validate the statement by checking on some examples)
- Crucial experience (to validate the statement by checking on a "crucial", difficult example)
- Generic example (to validate a statement by referring to an example, that is considered representative of a whole cathegory)
- Mental experiment (to validate a statement not referring to a given example, thus moving towards intellectual proofs).

The intervention tool is aimed at guiding students towards the proof construction. Moreover, the intervention tools aims at eliciting discussion on the necessity to move from pragmatic to intellectual proofs.

3. Design

3.1 Difficulties identified through the B2 questionnaire

The intervention tool aims at addressing specific difficulties that were outlined by means of Questionnaire B1 and B2 (questionnaire B1: questions 7-8-9-10-11; Questionnaire B2: Q2G1, Q2G2, Q2G3), namely difficulties in dealing with a geometric figure and its properties.

Moreover, the intervention tool is aimed at preparing student to the approach to proof.

This is the intervention tool for "avanced level": we recommend to address this intervention tool after having considered also tools referring to geometric domain and visuo-spatial cognitive domain and the "intermediate level" intervention tool .

3.2 Cognitive area and math domain of interest

The intervention tool refers to mathematical domain of geometry and the cognitive domain of reasoning, although there are relevant connections with the cognitive domains of memory (recovering geometrical facts and theorems) and visuo-spatial (dealing with a geometric figure, managing information in different representations including the visuo-spatial one).

3.3 Educational Aims

By means of the intervention tool, students are guided to construct a proof, by reflecting on important steps: understanding the text, identifying hypothesis and thesis, representing hypotheses on the figure and with other representation systems (such as algebraic formulas), recalling already known geometrical facts, organizing proof in form of a deductive chain of arguments.

The tool consists in a series of questions the teacher may pose to the students during a class discussion. Questions may be projected on the whiteboard. If the students have at disposal tablets or computers with internet connection, the questions can be administered by means of an interactive response system (e.g. Socrative, Mentimeter).

In this intervention tool we put into action specific guidelines of UDL.
Guidelines within Principle 1 (provide multiple means of representation), suggest proposing different options for perception and offering support for decoding mathematical notation and symbols.
The intervention tool offers guide and support for decoding a mathematical text.
Guidelines from Principle 2 (provide multiple means of action and expression) suggest to offer different options for expression and communication supporting planning and strategy development.
The intervention tool guides planning and strategy development.
Guidelines from Principle 3 show how certain activities can recruit students' interest, optimizing individual choice and autonomy, and minimizing threats and distractions. Students are asked questions in form of polls (which is the correct answer?) so as to promote their participation into the activity.

In terms of formative assessment, students care asked questions in forms of polls or open questions (strategy 5 : they become owners of their own learning); students are asked to give comments on incorrect answers of a fictitious student (strategy 4: they become resource for the others); after the poll, the teacher can promote a balance discussion (strategy 2); discussing the results of the poll the teacher an work individually or in small groups and, after each item or at the end of the activity, the teacher can promote a class discussion (formative assessment strategy 2). Students discuss their strategies and difficulties (strategies 4 and 5). The teacher can monitor students' progress throughout the game, giving feedback and prompts (strategy 3).

3.4 Addressing to Student /class

The intervention tool is addressed to all the class.

3.5 Educational activities: the Intervention Tool

The tool consists in a series of questions (in form of polls or open questions) the teacher may pose to the students during a class discussion. The questions are already put on a power point presentation, so that the teacher may project them on the whiteboard.
If the students have at disposal tablets or computers with internet connection, the questions can be administered by means of an interactive response system (e.g. Socrative, Mentimeter).

The power point file is provided in a separate attachment. Here we insert some comments on the sequence of questions.

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The students are provided the text of a statement to be proved. The text is accompanied by a figure.
First of all, the students are required to find the thesis in the text. The teacher can promote a discussion on students' answers (formative assessment strategy 2).

Draw an isosceles triangle ABC so that the base AB is smaller than the oblique side. Extend CA of a segment AE that is congruent to the difference between the oblique side and the base. Extend the base AB of a segment BF congruent to AE. Show that CF is congruent to EF	Find the thesis and highlight it

The same process can be followed for the hypotheses in the text.

Draw an isosceles triangle ABC so that the base AB is smaller than the oblique side. Extend CA of a segment AE that is congruent to the difference between the oblique side and the base. Extend the base AB of a segment BF congruent to AE. Show that CF is congruent to EF	Find and list the hypotheses

By means of the subsequent slide, the teacher can promote a discussion aimed at understanding how to "rabslate" the text into the figure. Students are also led to reflect on the fact that not all the hypotheses can be represented on the figure in an efficient way.

Draw an isosceles triangle ABC so that the base AB is smaller than the oblique side. Extend CA of a segment AE that is congruent to the difference between the oblique side and the base. Extend the base AB of a segment BF congruent to AE. Show that CF is congruent to EF	If you have to draw the figure, which is the first hypotheses you take into account?

Draw an isosceles triangle ABC so that the base AB is smaller than the oblique side. Extend CA of a segment AE that is congruent to the difference between the oblique side and the base. Extend the base AB of a segment BF congruent to AE. Show that CF is congruent to EF	If you have to draw the figure, which is the hypotheses that is the most difficult to represent on the drawing?

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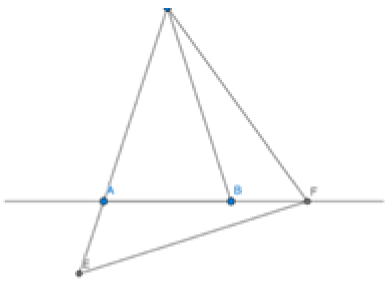
Draw an isosceles triangle ABC so that the base AB is smaller than the oblique side. Extend CA of a segment AE that is congruent to the difference between the oblique side and the base. Extend the base AB of a segment BF congruent to AE. Show that CF is congruent to EF

How can you represent all the hypotheses?

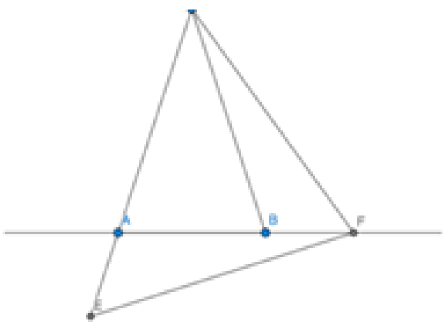
Draw an isosceles triangle ABC so that the base AB is smaller than the oblique side. Extend CA of a segment AE that is congruent to the difference between the oblique side and the base. Extend the base AB of a segment BF congruent to AE. Show that CF is congruent to EF

Hint: represent some hypotheses in form of equality, not on the drawing



Draw an isosceles triangle ABC so that the base AB is smaller than the oblique side. Extend CA of a segment AE that is congruent to the difference between the oblique side and the base. Extend the base AB of a segment BF congruent to AE. Show that CF is congruent to EF

Hint: represent some hypotheses in form of equality, not on the drawing



AE= AC - AB

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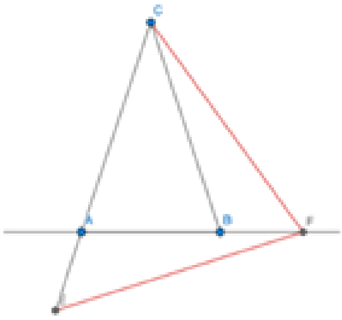
In the subsequent series of slides, students are asked to reflect on the thesis and led to understand that in order to reach the thesis one must recall the SAS criterium and apply it to the figure. This means to organize goal-oriented intermediate steps, working on the figure and on the equalities.
The teacher may also promote some polls to ask students to comment and evaluate answers given by fictitious classmates. In this way they act as resources for a fictitious classmate (strategy 4) and reflect on the importance of organizing proof as a discourse where statements must come from the hypothesis or from previous knowledge (intellectual proof).

Draw an isosceles triangle ABC so that the base AB is smaller than the oblique side. Extend CA of a segment AE that is congruent to the difference between the oblique side and the base. Extend the base AB of a segment BF congruent to AE. Show that CF is congruent to EF

What is represented by means of the red color?

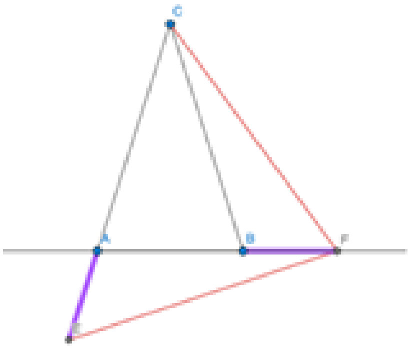
AE= AC - AB



Draw an isosceles triangle ABC so that the base AB is smaller than the oblique side. Extend CA of a segment AE that is congruent to the difference between the oblique side and the base. Extend the base AB of a segment BF congruent to AE. Show that CF is congruent to EF

What is represented by means of the purple color?

AE= AC - AB



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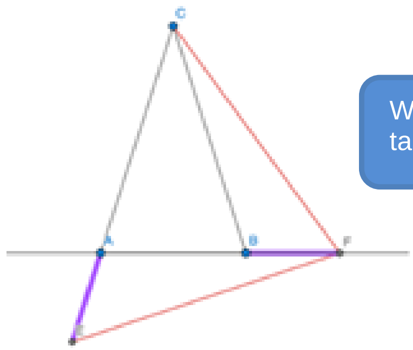
Draw an isosceles triangle ABC so that the base AB is smaller than the oblique side. Extend CA of a segment AE that is congruent to the difference between the oblique side and the base. Extend the base AB of a segment BF congruent to AE. Show that CF is congruent to EF

The goal is to prove that the two red segments are congruent.
You can take into account some triangles that contain the red segments and show that such triangles are congruent.



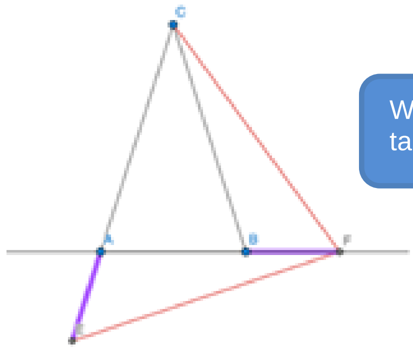
Which triangles would you take into account?

AE= AC - AB

Draw an isosceles triangle ABC so that the base AB is smaller than the oblique side. Extend CA of a segment AE that is congruent to the difference between the oblique side and the base. Extend the base AB of a segment BF congruent to AE. Show that CF is congruent to EF

Alice: triangles AEF and BFC

Barbara: triangles AFE and AEF



Which triangles would you take into account?

AE= AC - AB

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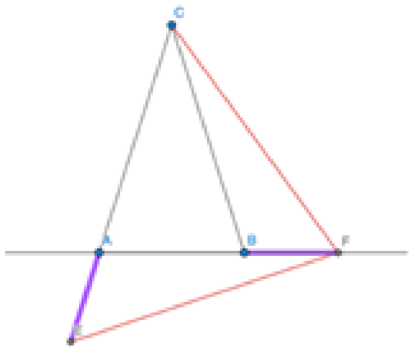
Draw an isosceles triangle ABC so that the base AB is smaller than the oblique side. Extend CA of a segment AE that is congruent to the difference between the oblique side and the base. Extend the base AB of a segment BF congruent to AE. Show that CF is congruent to EF

AE= AC - AB

The goal is to use the first criterion of triangles (SAS criterion).
I already know that AE=BF.
I need to show that segments AF and BC are congruent and that angles FAE and CBF are congruent.

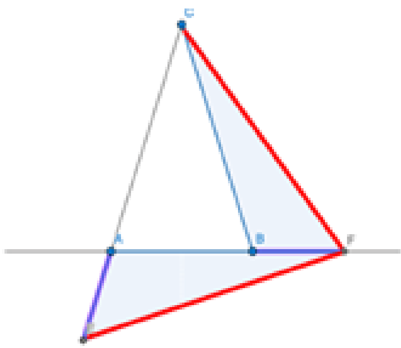


Draw an isosceles triangle ABC so that the base AB is smaller than the oblique side. Extend CA of a segment AE that is congruent to the difference between the oblique side and the base. Extend the base AB of a segment BF congruent to AE. Show that CF is congruent to EF

Consider the figures and the equalities. What about segment AF?

Recall that:
AE= AC – AB

AE=BF



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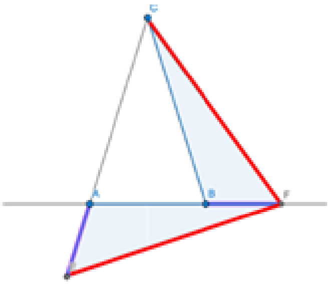
Since AE=AC – AB
You get
AB=AC - AE

You substitute the expression of AB into AF

AF= AB+BF=
AC-AE+BF=AC



Recall that:
AE= AC-AB

AE=BF

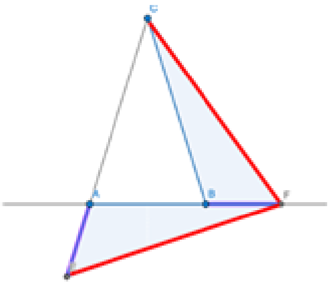
Draw an isosceles triangle ABC so that the base AB is smaller than the oblique side. Extend CA of a segment AE that is congruent to the difference between the oblique side and the base. Extend the base AB of a segment BF congruent to AE. Show that CF is congruent to EF

Since AE=AC – AB
You get
AB=AC - AE

You substitute the expression of AB into AF

AF= AB+BF=
AC-AE+BF=AC

What happened here? Look at the underscored segments

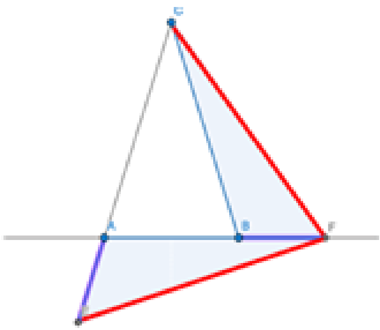


Recall that:
AE= AC-AB

AE=BF

Draw an isosceles triangle ABC so that the base AB is smaller than the oblique side. Extend CA of a segment AE that is congruent to the difference between the oblique side and the base. Extend the base AB of a segment BF congruent to AE. Show that CF is congruent to EF

You found that AF=AC



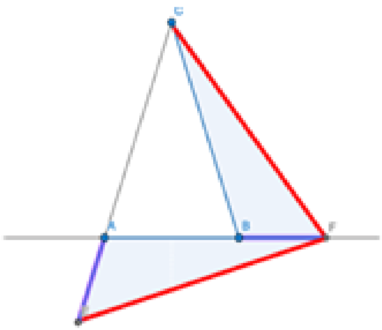
In order to get that AF=BC, which information can you use?

Recall that:
AE= AC-AB

AE=BF

Draw an isosceles triangle ABC so that the base AB is smaller than the oblique side. Extend CA of a segment AE that is congruent to the difference between the oblique side and the base. Extend the base AB of a segment BF congruent to AE. Show that CF is congruent to EF

You found that AF=AC



In order to get that AF=BC, which information can you use?

Recall that:
AE= AC-AB

AE=BF

Since the triangle ABC is isosceles, you have AC=BC

Then
AF=AC=BC
AF=BC

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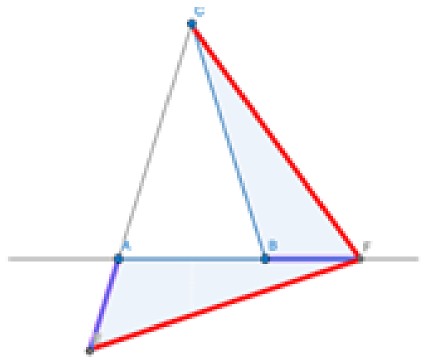
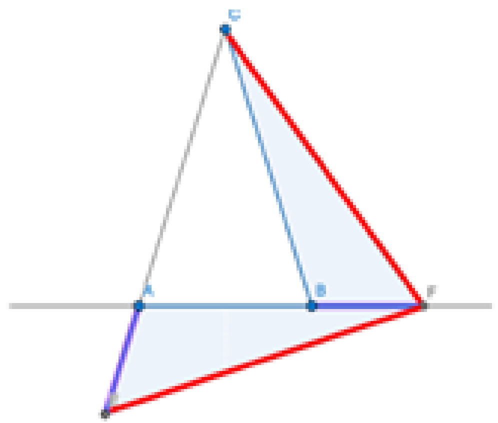
Draw an isosceles triangle ABC so that the base AB is smaller than the oblique side. Extend CA of a segment AE that is congruent to the difference between the oblique side and the base. Extend the base AB of a segment BF congruent to AE. Show that CF is congruent to EF



You showed that segments AF=BC.
Moreover, AE=BF.

In order to apply the SAS criterion, you need also…

Draw an isosceles triangle ABC so that the base AB is smaller than the oblique side. Extend CA of a segment AE that is congruent to the difference between the oblique side and the base. Extend the base AB of a segment BF congruent to AE. Show that CF is congruent to EF



Why are red angles (FAE and CBF) congruent?

Alice: by hypotheses

Claire: because they are supplementary to the same angle

Barbara: by construction

Darla: because they are supplementary to congruent angles

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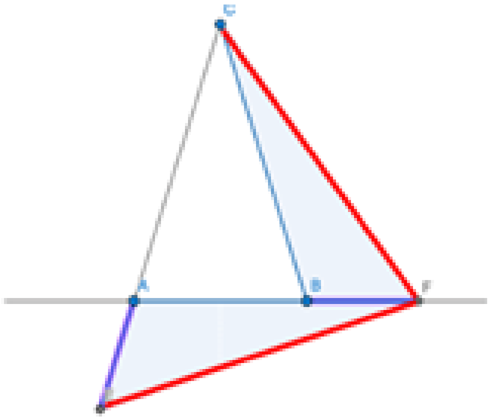
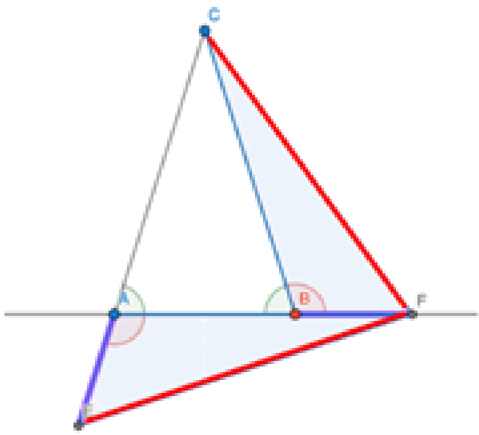
Draw an isosceles triangle ABC so that the base AB is smaller than the oblique side. Extend CA of a segment AE that is congruent to the difference between the oblique side and the base. Extend the base AB of a segment BF congruent to AE. Show that CF is congruent to EF



Darla: because they are supplementary to congruent angles

What are the congruent angles?
Why are they congruent?

Draw an isosceles triangle ABC so that the base AB is smaller than the oblique side. Extend CA of a segment AE that is congruent to the difference between the oblique side and the base. Extend the base AB of a segment BF congruent to AE. Show that CF is congruent to EF



Summing up:
AF=BC because of the equalities that are in the hypotheses and the fact that the triangle ABC is isosceles.

AE=BF by construction

Angles FAE and CBF are congruent because they are supplementary to congruent angles.

Then, for SAS criterion the light blue triangles are congruent

Then the segment CF and EF are congruent

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