Project number: 2018-1IT02KA201048274

Chapter 2

Analysis of the Learning Disabilities in Mathematics

Developed in the framework of the European project

SMiLD

Project Number: 2018-1¬IT02¬KA201¬048274

Co-funded by the Erasmus+ Programme of the European Union

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Introduction

Discomfort in the classroom and learning difficulties are manifestations often connected to risk factors, which could be of physical or socio-cultural origin, to be addressed with a theoretically founded and adequately elaborated intervention model based on empirical evidence, so as to avoid school failure. The different risk factors and their effects are attributable to some large-scale factors. According to numerous international researches, the influence of the socio-cultural background persists, above all, on the outcomes, which generates significant delays and distances in cognitive development, from the first years of life (Strand, 2014). In the category of learners defined as 'low achieving' (LA), the effects of deficits are more evident in the linguistic field, but important deficiencies are also found in the mathematical one (Geary et al., 2012). Low achieving students, those who fall below grade level, show frequent difficulties in operations linked to the memory process, thus solving problems requiring computational strategies as an effect of fact retrieval deficit. Limitations in problem-solving are often also linked to difficulties in reading and understanding texts, deficits in the formulation of hypotheses, in logical processes, in application and adaptation of math principles, and other different cognitive subtypes of mathematics learning difficulties (MLD) which have been classified through data-driven approaches (Bartelet et al, 2014). Motivational deficiencies also reduce the determination of pupils in tackling the cognitive obstacles posed by complex mathematical content. To cope adequately with these difficulties, it is necessary to propose methods that take into account the complexity of the factors that generate learning problems, the variety of their effects and the speed with which differences in skills are generated, so as to put in place targeted and multiple protective factors. It is a question of making early diagnoses adequate to identify the specific learning problems and of activating the cognitive processes of the pupils (such as perceiving, recognizing, conceiving, and reasoning) which are not adequately stimulated, with regard to the fundamental contents, in an integrated learning context of mathematics and language, which motivates them to success.

Low achieving in math even for those who present average literacy skills, have a direct effect on everyday life, resulting in fewer work opportunities and lower salaries as documented in analysis made in the UK and USA. Therefore, the importance of an educational system aware of this consequence is fundamental to tackle deficits in order to prepare young people to carry out, according to their own possibilities and choices, an activity or function that contribute to the material or spiritual progress of society.

1. MLD will be described illustrating how they manifest themselves and how can they be identified

Neurobiological basis could be the cause of math disability, which is considered a neurodevelopmental disorder, but math disability could also be a consequence of external factors. An evidence, thus negative, of how social environment affect the human body was already given in very ancient times by what is now called affective deprivation disorder, whose effects have been studied since the 1970s by Lytt Gardner in children's deprivation dwarfism or psychosocial short stature (PSS). Epigenetics, which is a very recent discipline, it is demonstrating with scientific evidence how socio-cultural factors can affect the organism and its functioning, to cause heritable phenotype changes, modifying the activation of certain genes, without altering the genetic code sequence of DNA. Since environmental experience modulates the levels and nature of epigenetic signals, they are considered fundamental in mediating the ability of the environment to regulate the genome. Epigenetics plays a fundamental role in all processes of neural reorganization or restructuring, including those that preside over brain plasticity. Crucial epigenetic changes are also involved in the regulation of learning and memory processes, environmental enrichment is also capable of curing learning and memory deficits. Therefore, multiple theories exist as follows:

- Core deficit hypothesis,
- Deficits in general domain hypothesis,

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- Deficits in domain-specific math areas,
- Procedural deficit hypothesis

These theories have their basis on dysfunction in specific brain regions mainly implicated in math processes. Furthermore, researches in educational psychology and general education supports the affective filter hypothesis, which is a concept relate to second-language learning theory and it regards a learning impediment triggered by negative emotional reactions to one's own environment. Some feelings, such as fear, anxiety and boredom, interfere with the learning process according to the affective filter hypothesis. Those negative emotions act as a filter between the speaker and the listener, reducing the amount of information that the listener can understand, thus preventing its efficient processing.

The terms used to describe students who experience problems with math, vary in the studies and regulations on the basis of the definition themselves of the target groups and according to the implementation of research instruments and tackling policy. The broadly used definition Mathematical Learning Difficulty (MLD) includes a wide variety of deficits, mostly affecting the area of arithmetic and thus arithmetic problem solving: generally speaking, MLD is used to refer to learning difficulties in all mathematical domains. *Mathematical difficulties* experienced by children depend on different factors varying from poor instruction to socio-cultural environment, with a broader meaning than the definition of math disability (MD). Not all students with mathematical difficulties will have MD, whose hypothetical paradigm refers to an inherent weakness in mathematical cognition independent from sociocultural or environmental causes. Therefore, since there are no standards to confirm the presence of learning difficulties (LDs) in math, the variations of the diagnostic criteria and different vision between the educational and medical systems responsible for taking care of these students must be taken in consideration as part of the teaching device.

The socio-cultural environment in which the students and the teachers are inserted, strongly influences the learning achievements, because a diagnosis of a disease instead of a difficulty depends on the respective official definition, thus radically changing the tackling perspective and testing procedures and reflects on the effectiveness of the efforts to improve the quality of the teaching/learning process. It is important, therefore for us to understand that the context influences both groups of the students and the teachers, so first of all we should identify the environment where teachers use terms like *dyscalculia* rather than *poor math achievement* to address children. The issue of a definition is still in progress, in Italy and most western countries the specificity of MLD diagnosis is included in a general category of 'Specific Learning Disabilities' (SLD) along with all the learning disabilities, students are therefore considered with 'Special Educational Needs' (SEN).

The World Health Organization International Statistical Classification of Diseases and Related Health Problems 10th edition (IChD-10) implements the classification of 'F81.2 Specific disorder of arithmetical skills', that "involves a specific impairment in arithmetical skills, which is not solely explicable on the basis of general mental retardation or of grossly inadequate schooling". Diagnostic tests, such as IChD 10 (WHO, 2003), or DSM 5 (2013), aim to identify subjects with Math disorders or Mathematical Learning disabilities as having specific difficulties in learning or in mathematics learning, based on medical models. Instead, others perspectives, such as European educational community, uses a broader concept of students with mathematical learning difficulties referring to any group of students with low achievement in mathematics (2013): "Low achievement is the situation where a child fails to acquire basic skills while they do not have any identified disability and have cognitive skills within the normal range. In those cases, low achievement may be considered as a failure of the education system".

"The deficit concerns mastery of basic computational skills of addition, subtraction, multiplication, and division (rather than of the more abstract mathematical skills involved in algebra, trigonometry, geometry, or calculus)". At the same time, the diagnostic guidelines raise the awareness that "Arithmetical disorders have been studied less than reading disorders, and knowledge of antecedents, course, correlates, and outcome is quite limited". We will use MLDs to refer to mathematical learning difficulties in all domains, which is to be considered multidimensional bringing into the picture mathematical domains other than the ones mentioned

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above. For example connected with Memory such as inhibition of irrelevant information from entering in the Working Memory; executive mechanisms connected with Reasoning such as Entailment; Inhibition (affective filter); Updating relevant information, shifting from one operation-strategy to another, Updating and strategic planning, Decision-making, Semantic memory; Visuo-Spatial Working Memory, and Visuo-Spatial reasoning/perception.

2. Which actions are to be carried out in teaching practice with reference to the Mathematical Learning Disabilities?

It is widely believed, among science and math teachers in particular, that many of the students' comprehension and learning difficulties depend on linguistic factors.

Knowing formulas by heart, however, is not enough in the face of the text of a problem, although the students remember them, sometimes they not seem to be able to recognize the question, interpret the instructions, identify the necessary elements to reach the solution, etc.

Sometimes, teachers also complain about children's difficulty in expressing: correcting math homework often requires interpretation and integration of disconnected and linguistically incorrect texts. The linguistic or textual organizational error, in a mathematics task, has to be considered serious, as if the knowledge of the contents could be independent of the ability to express them. The tools needed by teachers of scientific disciplines must be efficient to address language difficulties as a source of difficulty in mathematics.

In learning mathematics, communicative component is central, to express and transfer knowledge, skills, attitudes, experiences, which are continually reworked and intertwined; the learning path is therefore the result of a work in which language links different components to interact with each other.

Martha Fandiño analyses the aspects of learning process from another point of view, focusing on the strategies. She distinguishes among "conceptual learning", "procedural or algorithmic learning", "semiotic learning or management of representations", "strategic learning" and finally "communicative learning", which have decisive impact in the final phase of the learning process, when they switch to effective learning and final understanding. Communicative learning of math is an aspect of education that regards the ability to express logical ideas, telling, validating, justifying, arguing, demonstrating mathematical concepts (both orally and in writing) and visually representing them with figures.

The most important international survey in the framework of mathematical skills, the OECD-PISA survey, gives a definition of ‘Mathematics performance’ which, for PISA purposes, measures the mathematical literacy of a 15 year-old: “Mathematical literacy is an individual’s capacity to reason mathematically and to formulate, employ and interpret mathematics to solve problems in a variety of real-world contexts. It includes concepts, procedures, facts and tools to describe, explain and predict phenomena. It helps individuals know the role that mathematics plays in the world and make the well-founded judgments and decisions needed by constructive, engaged and reflective 21st Century citizens”.

For instance, students should be able to manage three mathematical processes:

- Formulating situations mathematically;
- Employing mathematical concepts, facts, procedures and reasoning; and
- Interpreting, applying and evaluating mathematical outcomes;

Linguistic competence plays a fundamental role. The frame of reference of the OECD-PISA survey explains this role and can be a very useful tool, also for teachers, to better define the phenomena and interpret the students' behaviours.

In the framework of the OECD-PISA survey, the importance of communicative competence is underlined in the three different aspects of mathematical processes – formulating, employing, interpreting. In the process of formulating language is fundamental considering reading, decoding and interpreting statements, questions, tasks in order to create a mental model of the situation; in the process of employing it is clearly stated that linguistic skills are necessary to articulate a solution, illustrate the work necessary to arrive at the solution and summarizing and presenting the intermediate results; finally, in the process of Interpreting, we need language to elaborate and communicate explanations and arguments in the context of the problem. Argumentative competence also manifests itself in the three phases of the problem-solving (modelling) cycle – formulate,

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employ, interpret and evaluate – in particular through providing, explaining and defending justifications for mathematical modelling, important for the acquisition of linguistic competence.

In light of all this, it is evident that it is necessary to build an interdisciplinarity link between the work of the language teacher and that of the scientific disciplines, and of mathematics in particular, in all school grades.

Since linguistic difficulties and disabilities actually interfere with the understanding of the text of a problem, the ability to identify the instructions, the possibility of finding an effective solution strategy, the ability to control the correctness and the sensibility of the result, the ability to justify the chosen strategy and to discuss and justify the final solution, the strategies to contrast language difficulties and disabilities should be taken into consideration to contrast as well the problems related to the role of language in learning mathematics. First of all the linguistical aspect of manuals: they should be balanced to reflect the age and the social environment of the students, furthermore it should be taken into consideration the use of other communication techniques, such as images, and other teaching strategies, on the one hand tackling learning disabilities, on the other hand strengthening of awareness of the potential of work on mathematical texts, for the improvement of language skills.

Among the motivational aspects that should be taken into consideration during the learning process, in all disciplines, and in particular in math, there is that regularity and harmony, for example in formulas and geometrical figures, are aspect related to the concept of beauty. Statements as "nice theorem", "nice proof" or "nice theory" are common among mathematicians: for a theorem, "beautiful" means short and clear, and in the case of a demonstration "beautiful" means not too short, because it refers to a well-expressed result. Harmonic and mathematical proceedings are strictly connected in art and music, as is evident for example in the golden ratio and musical harmonic scale, thus making students aware of how the characteristic features of "emotional" mathematical elements find themselves amplified and systematized in music and, more generally, in arts, should be implemented in school learning process. In the words of Leibniz: "Music is a hidden arithmetic exercise of the mind, which does not know that it is counting". Since learning styles may differ among students, real life examples stimulating different sensorial activities are considered extremely useful to tackle students' difficulties. Different anecdotes are related to the mathematical elements of the music theory, since Pythagoras and his disciples noticed that by vibrating two strings subjected to the same tension but of different length (1/2, 2/3 and 3/4 of the first one respectively), they obtained sounds that were particularly pleasant to the ear (consonants, in fact). It is the physiological structure of our hearing that makes us perceive the frequencies of sounds in a multiplicative rather than additive way: in short, with the ear we "count" in geometric progression, while with the fingers, adding units to units, we count according to an arithmetic progression. The scale is constructed from the fundamental frequency of a string taken as a unit and multiplied or divided by 3/2. Proceeding in this way, by ascending or descending fifths, multiplying by 3/2 or 2/3, we obtain the ratios of what is called the Pythagorean scale (although it actually dates back to Eratosthenes, in the 3rd century BC). For example, the note emitted by a string stretched by a quadruple weight has a double frequency: it will be said that it is one octave from the previous one and will be perceived as "equal", but more acute. The same observation can be repeated in terms of length: by shortening a string and in particular by pressing it to half its length and then pinching one of its halves, you will obtain a note at a higher octave. In today's piano keyboard, between two adjacent keys, black or white, there is an interval called "tempered semitone". The strings of any chosen semitone are in the same ratio. The scale produced according to the equable temperament is therefore obtained by dividing the octave into twelve equal parts. on a logarithmic scale. Since the octave is represented by the ratio 2:1, with a chain of simple proportions we obtain the value of the smallest interval, called temperate semitone, equal to the twelfth root of 2, which has a value of about 1.06, close to the diatonic semitone E-F which has a value of 256/243=1.053 and the value of the tempered tone, which corresponds to the product of two twelfth roots of 2, equal to 1.1224, therefore close to the value of the tone of the diatonic scale, that is 9/8=1.125 As we know today, the fundamental frequency (note) of the sound emitted by a tight string in vibration is directly proportional to the square root of the tension to which the string is subjected; it is inversely proportional to its length, the square root of its density and its section. This solution somehow saved the consonance of the intervals of the Pythagorean system and made the steps of the scale uniform, allowing composers and instrumentalists much more freedom and with more ability to play and compose, but it had to make use of the irrational, a concept rejected by Pythagoras because it denied the possibility of expressing any relationship by means of natural numbers. In the course of the 18th century, mathematicians better understood the nature of sound and were able to describe its propagation analytically. The French mathematician J.-B. J. Fourier came to the conclusion that – after Daniel Bernoulli (1700-1782), who believed he had described, by means of a trigonometric series, a particular type of sound – every periodic function can be expressed through a trigonometric series. Mathematical elements are present in the music of Arnold Schönberg (1874-1951), and his disciples, following atonality and dodecaphony, a compositional method that uses the twelve sounds of the chromatic scale free from reciprocal and hierarchical harmonic relations and reorganized, even with the use of combinatory techniques, according to the principle of the series. During the 19th century once undermined the principle of consonance/dissonance of musical chords, we have several examples of mathematical principles introduced into music with stochastic music based on Markov's chain theory, in 1955 Iannis Xenakis introduced probability into music: musical composition is processed by formal processes defined in probabilistic terms.

Imagination is, as well, an absolutely necessary element in mathematical thinking. It requires to be educated by a correct and fine interpretation of the language and rules though which mathematical objects are structured. It is possible to help a student in the right imaginative construction of an abstract mathematical question by choosing, in the history of mathematics, the paradigm closest to his/her cultural models and the most suitable to educate to think with a structure (build a model) that will remain consistent and functional even in further continuation of the study, when different branches, such as the geometric and algebraic ones, will mix together, for example in Zeno's paradoxes of Plato’s Socratic dialogues: In order to reach the turtle, Achilles should be able to travel a sum of infinite segments (and this is true); but the sum of infinite segments is not necessarily an infinite segment if they have zero length. Here it lies the paradox, indeed the false paradox: in order to execute sums of almost infinities of a distance of almost zero, the answer is not infinite. Zero and Infinity are two numbers like all the others, however, unlike the common ones, have some exceptional requirements: Zero, for example, multiplied by any number, always gives zero as a result, and Infinity, also multiplied by any number, can only give rise to infinity. What happens then when zero and infinity multiply? The result that emerges remains undefined. To understand that such sums can be finite, one had to wait until the 18th century. Only then it was started to put the basis for a rebellion against the diktat of the Peripatetic, which finally led Georg Cantor (1845-1918) to the creation of a satisfactory and coherent theory of mathematical infinity.

It is important to clearly express the meaning of the words through which a mathematical concept will be presented and the corresponding images that will be chosen to illustrate it: the construction of an abstract concept cannot be separated from examples. In mathematics an image can never be representative of the concept to which it refers, but simply serves to evoke it, nevertheless a wrong or misused image could lend itself more easily to misunderstandings than a poorly written text.

The word "imagination" is linked to the image, and provides the ability to create mental images, which are something certainly different from the figure seen on a book, or on a PC screen and is actually more abstract, for example, when it comes to those images that are the two-dimensional representation of a three-dimensional structure. The sense of sight is often connected with understanding, "I see" is, in many languages, under may circumstances synonymous of "I understand it": it does not necessarily refer to the sense of sight or to a real image, sometimes it can be a mental image or an abstract concept. Nevertheless, imagination could deal with all the five senses: this statement has to be taken into consideration when structuring different teaching strategies to reach the different students’ predispositions and abilities.

3. Analysis of the language used in Math text books in order to give evidence to the connection between language and student difficulties, identifying which language strategies are, or may be, used to better understand the notions and to have a better approach to solve problems.

Studies highlight how the difficulties of linguistic communication can make any type of direct intervention on mathematical contents vain. They stress also the fact that, for the teacher, they need to continually switch, during classroom work, from the use of languages to represent mathematics to languages to interact with the class that requires considerable metalinguistic awareness.

Beliefs about math in many cases affects motivational engagement of young students, with regard to textual problems, in particular those related to the formulation of the text of a problem, stereotypes (also linguistic) and misconceptions in the formulation of school problems induce erroneous beliefs and generate deviant attitudes towards problems and math itself. Critics about the use of the term 'misconceptions' have a theoretical foundation, and are the result of a progressive refinement of research in mathematical education. In particular, the idea of misconception and the approach to error is the starting point to a radical change that towards this definition which has put the student and their learning processes at the centre of attention. It is this shift of point of view where the learner is now considered as an active subject who builds his or her own knowledge. More precisely, this model undermines the traditional interpretation of errors. In fact, the student interprets the experience with mathematics, in particular the messages that the teacher continuously sends: the student gives meaning to these messages, a sense that naturally depends on the knowledge he has but also on many other less obvious elements. That algorithm, that term, that symbol, that property, that concept, will be internalized according to the sense attributed by the student, and it may happen that this meaning does not coincide with what the teacher intended to communicate. The learner, and more generally the individual, continually interprets the world, relating the observed facts with previous experiences: the beliefs are precisely the result of this continuous attempt to make sense of reality and, in the same time, determine the patterns with which the individual approaches the world and therefore interprets the future experience. In mathematical education, therefore, students' beliefs are seen as the result of their continuous process of interpretation of experiences with mathematics; on the other hand, in turn determining the patterns according to which future experience is interpreted, they act as a guide in selecting the resources to be activated; but, in particular, they can prevent from using adequate knowledge and resources. Beliefs and misconceptions act as a filter or as a simplification of a theory (as well as reality).

In the words of Lev Vygotsky:

> *"The scientific concepts evolve under the conditions of systematic cooperation between the child and the teacher. Development and maturation of the child's higher mental functions are products of this cooperation. Our study shows that the developmental progress reveals itself in the growing relativity of causal thinking, and in the achievement of a certain freedom of thinking in scientific concepts. Scientific concepts develop earlier than spontaneous concepts because they benefit from the systematicity of instruction and cooperation. This early maturity of scientific concepts gives them the role of a propaedeutic guide in the development of spontaneous concepts. The weak aspect of the child's use of spontaneous concepts lies in the child's inability to use these concepts freely and voluntarily and to form abstractions. The difficulty with scientific concepts lies in their verbalism, i.e., in their excessive abstractness and detachment from reality. At the same time, the very nature of scientific concepts prompts their deliberate use, the latter being their advantage over the spontaneous concepts. At about the fourth grade, verbalism gives way to concretization, which in turn favourably influences the development of spontaneous concepts. Both forms of reasoning reach, at that moment, approximately the same level of development".*

The example of the INVALSI Math tests administered in Italian schools in recent years have provided a great mass of results and highlighted many macro phenomena attributable to textual or linguistic difficulties of

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Italian students. As stated by Branchetti and Viale, the syntactic dimension of the mathematical text, does not seem to be particularly analysed in didactic studies, neither in mathematical nor in linguistic-educational ones. Only in recent years the opinion that the linguistic dimension represents a fundamental component of the study of mathematics has become increasingly solid, in contrast to a certain idea inherited from the school tradition, which wants language and mathematics separate and incommunicable areas.

Traditional math schoolbooks in Italy use problems which are often characterized by long periods with a complex syntax, the use of an implicit subordinate introduced from a past participle (e.g. given the trapezoid ...) or a gerund, verbal moods which are very frequent in mathematical texts. Typical of the traditional style of mathematics is also the use of the passive impersonal form (in Italian: 'si passivante') and of the parenthetical clauses. The high frequency of interpolated clauses which increases the information density of the sentence; the use of the form "so as to", which in Italian is followed by subjunctive mood, falls within the typical style of the traditional mathematical text . It is interesting to note that some textbooks reproduce syntactic modules typical of the Italian tradition of mathematical text of this genre also in the English version of some exercises, such as the following example from the English section "Test your skill" of a reference textbook used in a technical school during the first two-year of study: "Each day a company can produce a maximum of 300 tons of a certain product. For each ton produced the cost of manufacturing and raw materials is € 1,6 and the standing daily expenses are € 36,00. Find the maximum profit and the minimum amount so as not to be in deficit knowing that each ton is sold at € 4,00".

A good example of reference textbook is given in Italy by the work of Massimo Bergamini, Graziella Barozzi & Anna Trifone, *Manuale di Matematica blu*, *rosso*, *azzurro* and *verde*, edited by Zanichelli, which is aimed to a course that highlights the connections between mathematics and reality; the theory is expressed with particular attention to the use of a clear language, expressed with rigorous and precise criteria, and presents many exercises set in everyday life, a balanced use of images and references to activities linked to an online site. On the sides the formulas are represented with different system, to address students' different abilities and learning strategies and customs, and a part dedicated to those students with Special Educational Needs (SEN, in it. BES).

The Polish poet Wisława Szymborska dedicated several poems to mathematics, in her book *Wszystkie lektury nadobowiązkowe* (*Nonrequired reading*) on the prince of geometry theorems, she writes:

> *I can easily imagine an anthology of the most beautiful pieces of world poetry making room for Pythagoras's theorem. And why not? It sets off the sparks that are the mark of great poetry, its form is pared beautifully to only the most necessary words, and it has a grace with which not even every poet has been blessed…*

References

[1] J. Conway, P. Doyle, J. Gillman, W. Thurston, Geometry and the Imagination, in www.geom.uiuc.edu/docs/education/institute91

[2] Efraim Fischbein, The theory of figural concepts, in "Educational Studies in Mathematics" 24 (1993), pp. 139–162.

[3] Fandiño Pinilla M. I. (2014), *Diverse componenti dell'apprendimento della matematica*. In: D'Amore B. (Eds) "La didattica della matematica: strumenti per capire e per intervenire". Atti del Convegno Nazionale omonimo, 3-4-5 marzo 2014, Tricase (Lecce). Bologna: Pitagora, pp. 71-80.
Lytt I. Gardner, Deprivation dwarfism, in Scientific American 1972 July; 227(1): pp. 76-84.

[4] Stephen Krashen, (1982). Principles and Practice in Second Language Acquisition, online: http://www.sdkrashen.com/content/books/principles_and_practice.pdf

[5] E. Stevick, (1976) Memory, Meaning, and Method. Rowley, Ma.: Newbury House.

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The European Commission support for the production of this publication does not constitute an endorsement of the contents which reflects the views only of the authors, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

Strand, S. (2014). *School effects and ethnic, gender and socioeconomic gaps in educational achievement at age 11*. Oxford Review of Education, 40, (2), 223-225

[6] Geary, D.C., Hoard, M.K., & Bailey, D.H. (2012). *Fact retrieval deficits in low achieving children and children with mathematical learning disability*. Journal of Learning Disabilities, 45, 4, pp. 291-307

[7] Bartelet, D., Ansari, D., Vaessen, A. & Blomert, L. (2014). *Cognitive subtypes of mathematics learning difficulties in primary education*. Research in Developmental Disabilities, 35, 3, pp. 657-670

[8] Brian G. Dias, Stephanie A. Maddox, Torsten Klengel & Kerry J. Ressler, *Epigenetic mechanisms underlying learning and the inheritance of learned behaviors*, in "Trends in Neurosciences", 2015; 38 (2): pp. 96-107.

[9] Gottfried Leibniz, letter to Christian Goldbach, April 17, 1712. Original lat.: "Musica est exercitium arithmeticae occultum nescientis se numerare animi". In Gottschalk Eduard Guhrauer (Ed.): *Nachträge zu der Biographie. Gottfried Wilhelm Freiherr von Leibnitz*, Ferdinand Hirt's Verlag, Breslau 1846

[10] Wisława Szymborska, *Wszystkie lektury nadobowiązkowe*, Otwarte, 2015; English version: *Nonrequired reading. Prose pieces*, translated by Clare Cavanagh, Harcourt, New York-San Diego-London, 2002.

[11] Branchetti, Laura & Viale, Matteo. (2014) *Tra italiano e matematica: il ruolo della formulazione sintattica nella comprensione del testo matematico*. In Atti del Convegno internazionale "La didattica dell'italiano. Problemi e prospettive." Locarno 24-26 ottobre 2014, G.R.I.M. (Dipartimento di Matematica e Informatica, University of Palermo, Italy), 2014.

[12] Branchetti, Laura & Viale, Matteo, *Matematica e creatività linguistica: gli esercizi di stile applicati ai problemi aritmetici*, in "Opera Nuova, Rivista internazionale di scritture e scrittori", n. 19, 2019/1.

[13] Bradley, Renée, Danielson, Louis C. & Hallahan, Daniel P., *Identification of learning disabilities: research to practice*. Routledge, 2002

[14] Lev Vygotsky, *Thought and Language*, Cambridge (Massachusetts)-London (England), MIT Press, 2012, pp. 157-158.

[15] Parsons S, Bynner J & Brewer E. *Does numeracy matter more?* Natl Res Dev Cent Adult Lit Numer 2005;1-37

[16] Rivera-Batiz FL., *Quantitative literacy and the likelihood of employment among young adults in the United States*. J Hum Resour 1992;27: 313-28.

[17]Italian Constitution

[18] Giannis Karagiannakis, Anna Baccaglini-Frank & Yiannis Papadatos, Mathematical learning difficulties subtypes classification, in "Frontiers in Human Neuroscience", 2014; 8: 57.

[19] Neelkamal Soares, Teresa Evans & Dilip R. Patel, *Specific learning disability in mathematics: a comprehensive review*, in "Translational Pediatrics" 2018 Jan; 7(1): 48–62.

[20] Mazzocco MM. Defining and differentiating mathematical learning disabilities and difficulties. In: Berch D, Mazzocco MM, editors. Why Is Math So Hard for Some Children? The Nature and Origins of Mathematical Learning Difficulties and Disabilities. Baltimore, MD: Paul H. Brookes Pub Co, 2007:29-47.

[21] Mazzocco MM., *Challenges in identifying target skills for math disability screening and intervention*. In "Journal of Learning Disabilities" 2005 Jul-Aug; 38(4):318-23.

[22] World Health Organization. International statistical classification of diseases and related health problems. 10th ed. Geneva

[23] Bruno D'Amore, *Matematica. Stupore e poesia*, Giunti, 2010.

[24] Giorgio Bolondi, *Competenze linguistiche e competenze matematiche: interdisciplinarità e formazione degli insegnanti*, in F. Clementi & L. Serianni "*Quale scuola? Le proposte dei Lincei per l'italiano, la matematica, le scienze*, Carocci, 2015.

[25] Bolondi G., Fandiño Pinilla M. I. (2008). *Molteplici aspetti dell'apprendimento della matematica*. In. D'Amore B., Sbaragli S. (eds.) (2008). Didattica della matematica e azioni d'aula. Atti del XXII Convegno Nazionale: Incontri con la Matematica. Castel San Pietro Terme, 7-8-9 novembre 2008. Bologna: Pitagora. 129-131;

[26] Fandiño Pinilla M.I. (2008). Molteplici aspetti dell'apprendimento della matematica. Trento: Erickson.
OECD (2020), Mathematics performance (PISA) (indicator). doi: 10.1787/04711c74-en (Accessed on 12 May 2020).

[27] M. D'Aprile & P. L. Ferrari, *Linguaggi e rappresentazioni nella formazione degli insegnanti di matematica*, in "La matematica e la sua didattica" n. 4/2003.

[28] Rosetta Zan, *Difficoltà in matematica: Osservare, interpretare, intervenire*, Springer Milan, 2007.

[29] Zovkic, I. B., Guzman-Karlsson, M. C., & Sweatt, J. D., *Epigenetic regulation of memory formation and maintenance*, in "Learning & memory" (Cold Spring Harbor, N.Y.) 2013; 20(2), 61–74
Wisława Szymborska Wszystkie lektury nadobowiązkowe, Otwarte, 2015

Co-funded by the Erasmus+ Programme of the European Union